Primary mathematics: It's time we stopped teaching standard algorithms

Jerry Gehrke and Fred Biddulph

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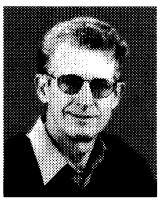

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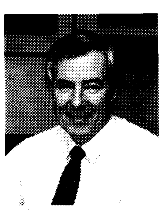

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Introduction

First, since in this article we focus on mathematics teaching at the primary level (years 1 to 8), by 'standard algorithms' we mean only the usual hand-written, column-aligned methods typically taught to primary children for adding, subtracting, multiplying and dividing with whole numbers and decimals.

Second, some questions:

- In your life outside the classroom, when you have to add, subtract, multiply or divide, do you calculate the answer using a hand-written method you probably learned at school, that is, a standard algorithm? Or do you use an electronic calculator? Or do you sometimes use your own methods, mental or written?
- What do other people you know do, at home and at work – use standard algorithms? Or a calculator? Or their **own methods**?
- Do you really **understand** all the standard algorithms you learned at school? Do you find them **meaningful**?
- Looking back, did you find learning the standard algorithms **fun**? Did you enjoy being told you had to add, subtract, multiply and divide in those particular ways? Do you think you might have liked being given the **freedom** to try to find your own ways to calculate?
- Did learning the standard algorithms tend to help you **like numbers**? Develop a good feeling for numbers, a good number sense? Develop your capacity to estimate answers

and to calculate mentally, using your own flexible methods?
- Did learning these tend to help you **like mathematics**? Appreciate what mathematics is about? Become a good mathematical thinker and problem-solver?
- Do you think all the time you spent learning these standard algorithms was time well spent? If so, do you think it would have been time well spent if there had been inexpensive, hand-held, electronic calculators around then, as there are today?
- And even if you think learning the standard algorithms was good for you, do you think it is good for most of the children in your class today (in the age of the calculator)?
- If it were just up to you (and you could have calculators available for the children in your classroom all the time, or

whenever you chose to do so) would you continue to teach standard algorithms? Or would you rather spend more time helping the children develop their own flexible mental and written methods of estimating and calculating, their ability to use calculators correctly, their ability to carry out mathematical investigations and solve problems requiring mathematical thinking, their understanding and appreciation of mathematics, and their mathematical creativity?

Concerns over continued teaching of algorithms at the primary level

Over the last twenty years or so, with the increasing availability of electronic calculators at home, at work and at school, mathematics educators around the world have increasingly questioned the wisdom of continuing to teach standard algorithms. For example, Plunkett (1979) contended that the increasing availability of calculators provided the opportunity to abandon the standard algorithms. About seven years ago Reys and Nohda (1994) summarised the situation:

A persistent theme of the current reform movement in school mathematics ... is the need to decrease the amount of emphasis on traditional paper-and-pencil arithmetic and develop a broader, more balanced approach to computation .... [This] is a challenge being addressed internationally.... (p.1)

Writing about primary mathematics education, Ritchhart (1994) was forthright:

A mathematics curriculum based on teaching the algorithms for computation is really no curriculum at all. (p.13)

... we must question the wisdom of continuing to devote so much of our time and energy to explicitly teaching these computational algorithms at all. We must examine to what extent this type of teaching may actually inhibit children's ability to think mathematically. (p.14)

With all modern societies in mind, Shumway (1994) wrote:

... a general question is to determine whether or not there is any *important role for paper-and-pencil algorithms in the twenty-first century. (p.192)*

More recently, Morrow (1998) asked several mathematics educators what the place of algorithms should be in a well-rounded curriculum. One responded:

In too many elementary school classrooms, far too much time is still spent on developing proficiency with pencil-and-paper algorithms. This reduces the time available for topics such as number sense. I believe strongly that the more inclusive ideas of number sense – estimation, mental math, a "feel" for large numbers – are more important than algorithmic proficiency and that they need to be given more time and attention in an effective mathematics program. (p.5)

Another said:

We need to shift our emphasis from a curriculum heavy on algorithms and from the view that all students should become adept at handling algorithms before tackling some of the more interesting aspects of mathematics to a curriculum that explores mathematics in all its wonderful variety. (p.6)

Recently one of us visited a primary classroom in which the teacher had set her nine to ten-year-old children the task of using the decomposition subtraction algorithm to complete a worksheet of exercises all of the kind

$$\begin{array}{r} 46 \\ -\underline{28} \end{array}$$

What a terrible waste of children's mathematical thinking power! If anything the children should have been doing these in their heads, indeed were able to do them in their heads, as the author found when talking to several of the children. These children found it far more sensible and meaningful to say, "Well, 2 makes 30 and another 16 makes 46, so that's 18 altogether."

"In other words, such algorithms have not been part of the mathematics requirements for our five to ten-year-old children for almost ten years. "

To further illustrate our point about children's power of thinking in mathematics – when they are not shackled by the use of standard algorithms - we make further use of our example 46 – 28. Some children have figured out that 40 – 20 is 20, and that 6 – 8 is 'minus 2' (that is, negative 2), so 2 less than 20 is 18. Had we stuck with the usual algorithm you can guess what we would have been telling the children, namely, "6 minus 8, you can't do that, so...". But, of course, this is nonsense; you **can** do "6 minus 8" if you use negative numbers, as some children have discovered for themselves.

It is interesting that standard algorithms continue to be taught in New Zealand primary school classrooms even though there is **no mention** of them in our mathematics curriculum document at Levels 1 to 3 (Ministry of Education, 1992). In other words, such algorithms have not been part of the mathematics requirements for our five to ten-year-old children for almost ten years. On the other hand, there is in our mathematics curriculum document at Levels 1 to 3 considerable reference to performing mental calculations, explaining the meaning of numbers, estimating and problem solving.

The writers of the New Zealand mathematics curriculum document were soon vindicated in this decision by publication of the findings of classroom-based research in the United Kingdom by Shuard (for example, Shuard, 1992) and in the U.S.A. by Kamii, Lewis and Livingstone (1993). Shuard's research showed that allowing young children to use calculators whenever they liked and not teaching them standard algorithms had no negative effect on their mathematical learning. The research by Kamii, Lewis and Livingstone clearly showed the limitations placed on children's mathematical thinking by a steady diet of standard algorithms. For example, they had teachers in several elementary classes at Hall-Kent School ask their children to carry out two calculations, namely

$$\begin{array}{r} 504 \\ -\underline{306} \end{array}$$

and 13 x 11 (with the latter to be done mentally). The results are summarised in Tables 1 and 2 below.

children still need to learn:

- that numbers are ideas (of quantity) and numerals are names for numbers;
- what the operations of adding, subtracting, multiplying and dividing **mean**, including the relationships amongst them (for example, that subtracting is the opposite, or inverse, of adding; that multiplying is repeated adding);
- how our Hindu-Arabic numeration system works (that it is a base-ten, place-value system with ten symbols, including a symbol for zero, and that it is written horizontally with place values in descending order left-to-right); that we read our numerals left-to-right;
- the basic adding and multiplying facts (that is, for adding, the sums from 0+0 to 9+9 and all the sums in between; similarly for multiplying), together with the corresponding subtracting and dividing facts;
- how to use these basic facts to work out, for example, 40+70, or 6x80;
- how to determine for any given simple real-life question involving numbers, what calculation, or sequence of calculations, can be done to find the answer to the question;
- how to find reasonable approximate answers to calculations (for example, that 38x124 is very approximately 40x100 or 4000).

Second, we need to realise that to teach primary mathematics well in the age of the calculator we need, if anything, to be better teachers, and have a better understanding of mathematics, than before. We need to be ready and able to set our children free (mathematically), and to cope with the delights, often unexpected, that follow. In short, we may need to develop a new pedagogy for teaching mathematics, one that involves turning our classrooms into communities in which everyone is a learner, a doer, and (potentially) a teacher of mathematics, in which everyone communicates about, and helps one other with, their mathematical endeavours. This would be to take a social constructivist approach to teaching and learning (Biddulph & Carr, 1999), as did the grade 2 and 3 teachers at Hall-Kent School. What we as teachers of mathematics should aim to achieve is to help the children in our classes become mathematical thinkers, questioners, creators and problem-solvers. In short, we should aim to help them become little mathematicians, to the best of their abilities. We will come much closer to achieving this aim in primary classrooms by **not** teaching standard algorithms than we will by teaching them.

Table 1: Results for the subtraction exercise

Grade	No. classes	Taught the algorithm?	% children correct
2	2	No	74%
3	1	No	80%
4	4	Yes	29%, 38%, 39%, 55%

Table 2: Results for the mental multiplication exercise

Grade	No. classes	Taught the algorithm?	% children correct
3	1	No	60%
4	4	Yes	5%, 6%, 14%, 15%

So, grade 4 children (mostly ten-year-olds) who had been taught the usual subtraction and multiplication algorithms did much worse on these two tasks than younger children (mostly eight and nine-year-olds) who had not! The younger children showed much greater number sense on both tasks.

Issues

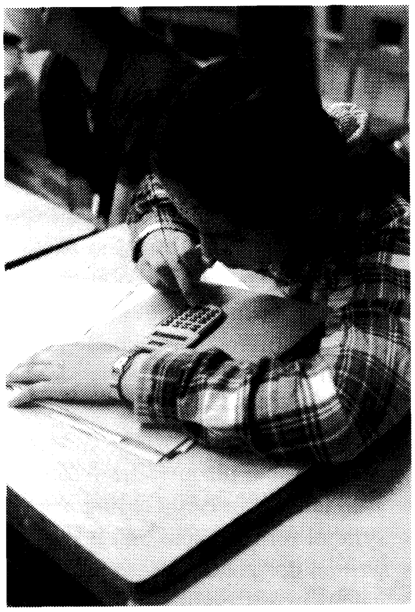
In advising that we should stop teaching standard algorithms in our primary classrooms, we are aware that there are important issues to sort out. First, we need to think carefully about what else we should and should not change regarding our teaching of number. In our view, we still need to teach, and

Concluding comments

Our standard algorithms have been around for four or five hundred years (Usiskin, 1998). They were, by and large, the best calculating 'technology' for the times, and they gradually supplanted the use of more cumbersome, though quick, counter-reckoning technologies (Barnett, 1998). However, for twenty years or so we have had a new calculating technology, the electronic calculator, that is faster, no less readily available, no more cumbersome, and much easier to learn to use than these paper-and-pencil algorithms. The electronic calculator has largely supplanted standard paper-and-pencil algorithms in the world outside school, and, in our view, it is time we stopped teaching these algorithms in school, at least at the primary level. The benefits should be enormous.

For one thing, we should then be more inclined, and have more time and opportunity, to help children really think about the mathematics they are doing and realise that mathematics makes sense. We should be better placed to engage children in real mathematical activities, including investigating, detecting and creating patterns, and posing and solving problems, rather than having them do seemingly endless and meaningless algorithm exercises on worksheet after worksheet.

It's time to make a start. We hope you will talk about this with your colleagues. You might do a little further reading around this issue. You might carry out a little action research in your own classroom to see what happens when you don't teach standard algorithms and instead let children find their own methods and use the calculator when they want to. You would then be following what we consider to be the wise pedagogical advice given in an address to the American Institute of Instruction in 1830 by Warren Colburn, (as quoted in McIntosh, 1998, p.48):



> *The learner should never be told directly how to perform any operation in arithmetic ... Nothing gives scholars so much confidence in their own powers and stimulates them so much to use their own efforts as to allow them to pursue their own methods and to encourage them in them.*

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