Helping Children Move beyond counting to part-whole strategies

Jenny Young-Loveridge

> *"Learning to count is an key aspect of children's early numeracy learning Not only is counting an important* ***procedural*** *skill used to determine quantity, but counting is also important for helping children build a solid* ***conceptual*** *understanding of the number system."*

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Introduction

Learning to count is an key aspect of children's early numeracy learning (Young-Loveridge, 1987, 1991). Not only is counting an important *procedural* skill used to determine quantity, but counting is also important for helping children build a solid *conceptual* understanding of the number system (Young-Loveridge, 1991). Research on children's counting errors shows extensive understanding about how the number system is structured (Young-Loveridge, 1987).

It is important to distinguish between *Rote Counting*, reciting number names in the correct order, and *Enumeration*, a process which involves using the sequence of number names in one-to-one correspondence with a group of objects to work out the answer to a "How many?" question (Young-Loveridge, 1987). According to some writers, if children are to use counting successfully to determine quantity, they need to understand three basic principles which govern counting (Gelman & Gallistel, 1978). The one-to-one principle is about maintaining one-to-one correspondence between the number names in the counting sequence and the objects being counted. The stable order principle is about producing the number names in a consistent order. The cardinality principle is about understanding that the last number name in a counting sequence tells how many objects are in the group have been counted (ie, the answer to a "How many?" question). Two other principles include the order irrelevance principle, an understanding that objects can be counted in any order, and the abstraction principle, an understanding that any objects can be counted, even numbers themselves (Gelman & Gallistel, 1978). The mathematics curriculum document includes many references to rote counting and enumeration in the Number strand at level 1 (Ministry of Education, 1992).

Counting as part of a Developmental Progression

Researchers who have explored children's strategies for solving addition and subtraction problems have noted that initially these often involve some kind of counting (eg, Carpenter & Moser,1984; Wright & Gould, 2000). Carpenter and Moser identified a developmental progression in children's strategies for addition and subtraction between grade 1 and grade 3, as they moved from "Counting All" to "Counting On," and then to "Number Fact Retrieval". The *Count Me In Too* programme has a similar hierarchy of strategies within its framework for early number development (see New South Wales Department of Education & Training, 1999). After "Emergent Counting," comes "Perceptual Counting" (counting perceived objects), then "Figurative Counting" (counting concealed objects by counting mentally), both processes that involve Counting All from One. This is followed by "Counting On," then "Facile Number Sequence" – somewhat of a misnomer because one of its key features is the use of strategies other than *number sequence.* It is in the "Facile" stage that children use part/whole strategies to solve number problems.

Part/Whole Relationships

There seems to be reasonable agreement that, although counting provides an important first step towards understanding numbers, the emphasis on counting must shift to a focus on part/whole relationships among numbers, if a more advanced understanding about numbers is to be achieved (Baroody, 1990; Cauley, 1988; Fuson et al, 1997; Van de Walle, 1990). Van de Walle argues that if "instruction moves directly from counting to the introduction of symbolic addition and subtraction...many, if not most, children fail to develop number sense characterised by a rich variety of relationships. Instead they continue to count their way through elementary school" (p. 86). The "part-whole" model has an advantage over the "mental number line" model of counting, in allowing children to think about relationships among numbers in much more complex and flexible ways (Resnick, 1983). The "part/whole" idea makes it possible to think about numbers as compositions of other numbers, with the combined parts neither exceeding nor falling short of the whole. Many writers have stressed the importance for children of coming to understand the "additive composition of numbers," and recommend giving children lots of experiences with single-digit sums and difference to 18. Children need to understand that there are many ways to construct a particular number (eg, 18 can be from 9 and 9, 10 and 8,11 and 7, and so on). According to Resnick (1983) "the major conceptual achievement of the early school years is the interpretation of numbers in terms of part and whole relationships" (p. 114).

Evidence to support the value of a curriculum which emphasises part/whole relationships comes from a study by Fischer (1990). She found that kindergarten (five-year-old) children who had been given lots of experiences which stressed set-subset relationships went on to develop a more mature concept of number, were more successful in solving addition and subtraction word problems, and developed greater understanding of place value than a comparable group of five-year-olds who received standard instruction on number concepts, using a "count/say/write" approach which emphasized counting by ones.

Unfortunately, some children do seem to get stuck on counting, and find it hard to make the transition to part/whole strategies. As school mathematics becomes increasingly challenging, their dependence on counting is likely to be more and more of an obstacle to their mathematics learning. In a recent study with Year 6 children from a low and a medium decile school (n = 132), I found that about 15% of the children showed no evidence of part/whole strategies (Young-Loveridge, 2001). This was of considerable concern, given that these children would soon be going to an Intermediate School to begin Year 7.

Te Maunga Tau (The Number Mountain): NZ Number Framework

New Zealand has developed its own framework for number learning, known as Te Maunga Tau (The Number Mountain) based partly on the work with *Count Me In too* (see Ministry of Education, 2001a, 2001b, 2001c; Wright, 2000) (see Table 1).

Table 1
Stages in the NZ Number Framework (Te Maunga Tau): The Number Mountain

Counting Strategies	
0	Pre-Counting
1	Count All from One
2	Advanced Counting (Counting On)
Part/Whole Strategies	
3	Early Additive
4	Advanced Additive
5	Advanced Multiplicative
6	Advanced Proportional

The framework consists of three stages which involve increasingly sophisticated counting skills (Pre-Counting, Count All from One, and Advanced Counting; ie, Counting On), then four stages which involve the use of increasingly complex part/whole strategies (Early Additive, Advanced Additive, Advanced Multiplicative, and Advanced Proportional). Children at the lowest level on the framework, Pre-Counting, are unable to count, either because they don't know the sequence of number names, or they have problems maintaining one-to-one correspondence between the number names and the objects being counted. At the next stage, Count All from One, they can count, but their counting begins at one. Initially, they are able to count only if materials are available (ie, Perceptual Counting), but eventually they can count mentally, without needing objects (ie, Figurative Counting). In the next stage, Advanced Counting, the major advance is in the ability to Count On from one of the addends, rather than Counting All from One. At this stage, children are able to recognise that when a number name is used to refer to a collection of objects, it implies a counting sequence up to that number name, and hence does not need to be repeated (Wright, 2000). Counting On is the most advanced of the counting strategies and provides the stepping stone to a completely different kind of strategy, Part/Whole strategies.

Part/whole strategies involve splitting numbers into parts (ie, partitioning) and joining the parts together in ways to solve problems without the need to count. At the simplest level, Early Additive Part/Whole, the splitting and joining of numbers involves only one or two splits. For example, 9 + 8 could be solved by splitting the 8 into 7 and 1, combining the 1 with the 9 to make 10, and adding the remaining 7 to the 10 to make 17 altogether. At the Advanced Additive Part/Whole level, the numbers are larger and there are more splits. For example, 15 and 27 could be solved by splitting the "15" into 12 and 3, combining the "3" with "27" to make "30", then adding on the 12 to make a total of 42. At this level, children can choose appropriately from a wide range of different strategies to solve problems that involve addition and subtraction. Children at this stage can see numbers "as whole units in themselves, but also 'nested' within these units are multiple possibilities for subdivision" (Ministry of Education 2001a, 2001b, p. 4 of Section A). Children at the Advanced Multiplicative stage can choose appropriately from a range of strategies that involve multiplication and division. Those at the Advanced Proportional stage can choose appropriately from a range of strategies that involve fractions and proportions.

Another important aspect of *Te Maunga Tau* is the distinction between *Strategies* and *Knowledge*. Strategies are the ways that children solve number problems, in particular, the mental processes they use. Knowledge includes they key information which children need to have in order apply particular strategies. These are seen as mutually supportive, with *strategies* and their use leading to the creation of new knowledge, and *knowledge* providing the foundation for strategies.

Numeracy Development Projects

The publication of results from the Third International Mathematics and Science Study (TIMSS) in the mid to late nineties, showed relatively low levels of mathematics achievement for children in Western nations (compared to Asians), and has contributed to a world-wide focus on numeracy (see Commonwealth of Australia, 2000; Garden, 1996, 1997; National Council for Teachers of Mathematics, 2000; Ministry of Education, 1997; Reynolds, 1998). In 1998, the Ministry of Education began work on a comprehensive numeracy policy and strategy for New Zealand (see Ministry of Education, 2001a). In 2000, the National Administration Guidelines (NAGs) were modified, requiring schools to give priority to numeracy as well as literacy (see Ministry of Education, 2000). Two major numeracy projects were undertaken in 2000 to improve teachers' professional knowledge, skills, and confidence. These included the Count Me In Too Pilot Project (years 1-3) and the Exploratory Study (years 4-6). In 2001, three projects, one for early numeracy (years 1-3), one for advanced numeracy (years 4-6), and an exploratory study (years 7-10), got under way at the beginning of the year. More than 3,000 teachers and 60,000 children participated in one of these three projects. Key elements of the projects include the framework for early number learning (described above), a strong focus on the mental strategies used by students to solve number problems, and a professional development programme for teachers to help them identify a child's stage on the number framework so that his or her learning needs can be more effectively met. A working definition of what it means to be numerate as an adult has been developed as part of the work for the Numeracy Strategy. According to the definition, a numerate person has "the ability and inclination to use mathematics effectively in [his or her life] – at home, at work, and in the community" (Ministry of Education, 2001a, p. 1).

An evaluation of the Count Me In Too Pilot conducted in 2000 showed impressive results, with clear and positive growth in all five aspects of number learning measured, regardless of age, ethnicity, region, or decile (see Thomas & Ward, 2001). The aspect of number learning which is particularly relevant to this paper is the Stage of Early Arithmetic Learning (SEAL). Initially, approximately one quarter of the seven- and eight-year-olds used part/whole strategies. By the end of the project, this proportion had increased to just over half. However, Thomas and Ward expressed their concern that more than 40% of the children in this age group were still not displaying part/whole strategies. Systematic differences between ethnic groups were found in the use of part/whole strategies. Among the seven-year-olds, there were significantly fewer Polynesian students (Maori – 42%; Pacific Islands – 38%), than European or Asian students (60% & 64%, respectively) using part/whole strategies by the end of the project.

Addition & Subtraction across a Decade Break

In a recent study with Year 6 children, I found that part/whole strategies were used much more frequently for addition problems than for similar subtraction problems. Just over half the children used a part/whole strategy for 8 + 7, and 9 + 4, whereas a third used part/whole strategies to solve 14 – 9. When the problem involved two-digit subtraction with re-naming, even fewer children used part/whole strategies, with just under a quarter (23%) using part/whole strategies to solve 53 – 26. A further 14% solved the problem by imagining the written algorithm, and three children successfully counted backwards, making a total of 39% who got the correct answer. This success rate is only slightly higher than for younger children doing a similar problem for the National Educational Monitoring Project (NEMP) at Year 4 (34%), and is substantially less than for those at Year 8 (80%) (see Flockton & Crooks, 1998). The most common *incorrect* strategy used by the Year 6 children in my study was to imagine the problem in working (vertical) form, and subtract the smaller digit from the larger digit, irrespective of its position (14% got an answer of 33 by subtracting 2 from 5 and 3 from 6). Of the 30 children who used part/whole strategies, only one turned the problem into a missing addend problem, going up from 26 to 53 in jumps of 4, 20, and 3. Two children used a compensation strategy, beginning by subtracting 30 from 53, then adding 4 back on. Five children subtracted 20 from 53, then took away 6 by subtracting first 3, then another 3. Eight other children used a similar strategy, subtracting 20 from 50, then taking away 3. Six children began by subtracting 6 from 53, then took away 20. Eight children used their knowledge of doubles (ie, 25 + 25 = 50) as a starting point for solving the problem, making the adjustments necessary for 53 less 26.

Although 85% of the children showed evidence of using part/whole strategies for at least one problem, only about a quarter of these children did so for two-digit subtraction with re-naming. These results point to the need to strengthen children's part/whole understanding and help children learn to use part/whole strategies across a range of different problem types. Many writers advocate the use of manipulative materials to aid the understanding of concepts.

Mental Actions

Koeno Gravemeijir (1994), a mathematics education researcher from the Netherlands, has written about the idea of "mental actions" and their relationship to "manipulative actions" (ie, actions done with manipulative materials). According to Gravemeijir, what begins as a manipulative action is internalised and eventually becomes a mental action. He argues that the manipulative action must correspond exactly to the mental action. He writes about judging the usefulness of manipulatives in terms of their match with the intended mental activity. He shows how some manipulatives do not meet this requirement, highlighting the discrepancy between a manipulative action and the intended mental action. As children develop counting strategies, they move from "Counting All from One," where the mental action required is counting and recounting, to "Counting On" which involves counting on from one addend using a double counting process. The double counting is necessary to keep track of the number of objects altogether, as well as the number of objects from the second addend which have so far been counted (eg, double counting for 5 + 4 involves "six – one, seven – two, eight – three, nine – four"). Later the mental action may change to recalling a memorized fact, or to deriving a number fact, by splitting and joining numbers from a memorized fact. Eventually children can let go of thinking about concrete materials and can work with number relationships in an abstract way. Below is a description of the process whereby three-dimensional tens frames can be used to move children's thinking away from counting to splitting and joining numbers to make "Tidy" Tens".

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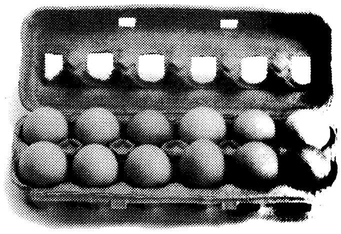
Three-dimensional Tens Frames

Tens frames, in the form of a five by two array, are suggested in several resources designed to enhance children's numeracy (see Ministry of Education, 2001b, 2001c; Young-Loveridge, 1999a, 1999b). The quinary structure of the tens frame, based on groupings of five (eg, 6 as 5 + 1, or 8 as 5 + 3), can make computation much easier. To create tens frames, the copy masters provided for this purpose are photo-copied onto cardboard, then cut and laminated, for use with coloured counters (one per compartment). A disadvantage of these tens frames is that the counters slide easily across the laminated surface, and a slight jolt can disturb the arrangement of counters on the tens frame. A three-dimensional tens frame with a separate compartment for each object eliminates the problem of objects sliding away. An empty egg carton can be used to create such a tens frame which, because it is three-dimensional, prevents the contents of the compartments from moving.

Converting empty egg cartons to tens frames is easy, although with several different kinds of cartons, slightly different modifications are required. With a "one dozen" carton, I start by trimming off the lid and side flaps. I then cut the last pair of compartments off with scissors and trim diagonally across the cut corners so they are rounded off. With a 30-egg tray (six rows of five), I use a craft knife to cut along the ridge between the second and third row of six, and between the fourth and fifth row, so that I get three tens frames from a 30-egg tray. Objects such as pebbles, blocks, or other small objects that are easy to grasp and heavy enough to stay securely in the compartments seem to work well. Recently I have begun using multi-link cubes, choosing a different colour for each of the two addends. This helps to distinguish one addend from the other.



Tidy Tens

The idea of "Tidy Tens" came from some work I have been doing in a Year 5/6 class where the focus was on "basic facts" (single-digit addition problems). Three-dimensional tens frames were given to the children and they were instructed to make each addend in a separate tens frame, using blocks of one colour for the first addend, and another colour for the second. We talked about the importance of having "Tidy Tens," and looked at ways that we could move blocks from one tens frame to the other to create a "Tidy Ten." The blocks that were left behind in the other tens frame were called "Leftovers." For example, if there are 9 objects in one tens frame and 8 in the other,

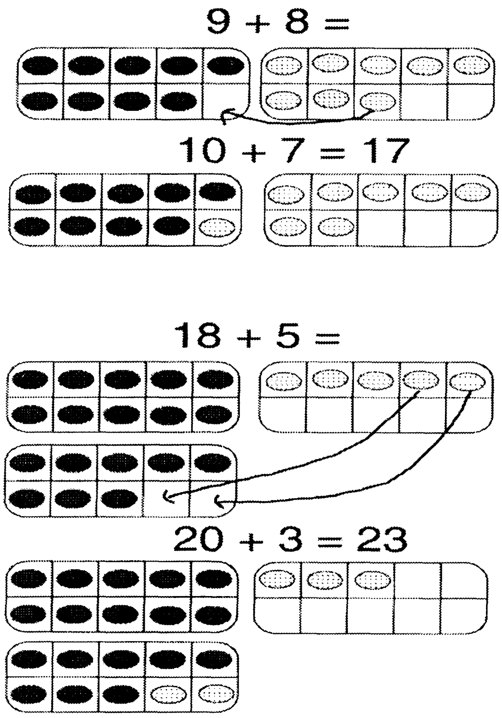


Figure 1
Part-whole strategies used iwth ten-structured materials to solve "9+8" and "18 + 5": "Tidy Tens" with "Leftovers"

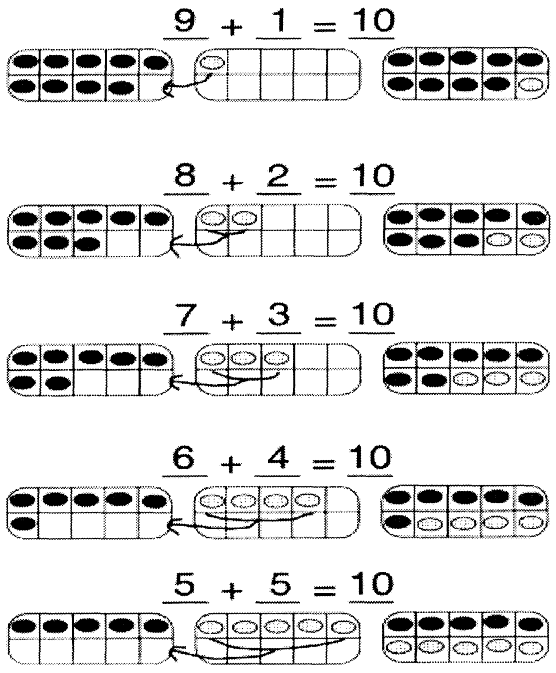


Figure 2
Combinations which make ten: "Tidy Tens" with no "Leftovers"

one object can be taken from the 8 and put with the 9 to make 10, leaving 7 behind in the other tens frame (see Figure 1). Joining 10 and 7 becomes very easy once children understand how a whole decade can be joined with a single-digit quantity (eg, 10 + 7 = 17, 20 + 8 = 28). An alternative way of making a "Tidy Ten" is to take 2 from the 9 to put with the 8 to make 10, leaving 7 behind in the other tens frame. It is important for children to see that either strategy works, but one strategy usually involves fewer adjustments, and this strategy is the more efficient of the two.

After we had established the process for making a "Tidy Ten" we looked at various single-digit combinations to identify which pairs of numbers make a "Tidy Ten" with *no* "Leftovers" (see Figure 2). Knowledge of the single-digit sums for ten is vital for children using part/whole strategies.

Once the children had understood the concept of "Tidy Ten" using two single-digit quantities, they could begin working with problems involving larger addends, using additional tens frames (for an example, see Figure 1). It is important that the sum of the two quantities is beyond the next decade break, in order to help children understand the processes involved in multi-digit addition and subtraction with re-naming. Below are examples of increasingly complex problems which build on the understanding of "Tidy Tens" for two single-digit numbers together totalling more than 10:

- a "-teen" number plus a single-digit quantity, together totalling more than 20
- two "-teen" numbers, together totalling more than 30
- a "twenties" number plus a single-digit quantity, together totalling more than 30
- a "twenties" number plus a "-teen" number, together totalling more than 40
- two "twenties" numbers, together totalling more than 50

Recording the Splitting and Joining Processes

Once children are comfortable with modelling the operation using with manipulative materials, they can move to a recording system which is more abstract. This way of recording shows the splitting of one quantity in such a way that one of the parts can be joined with the other addend to make a "Tidy Ten" (see Figure 3). In the example of 9 + 8, the 8 is split into 1 and 7. The 1 is then joined with the 9 to make a "Tidy Ten," shown by a ring around the combination. Rewriting the problem as 10 + 7 may be helpful for some children who need to see this intermediate step. Once the answer of 17 is recorded, the children can be asked to explain the relationship between each,

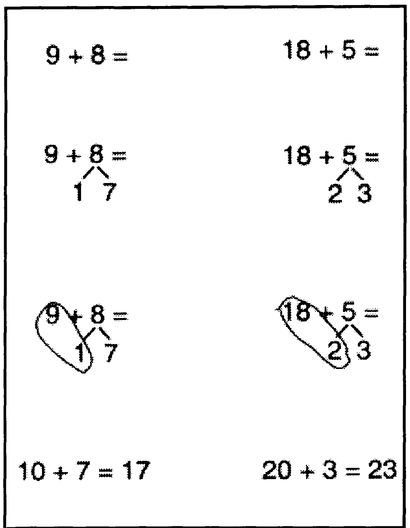


Figure 3
Method for
Recording Addition using
Part/WholeStrategies of
"9 + 8" and "18 + 5"

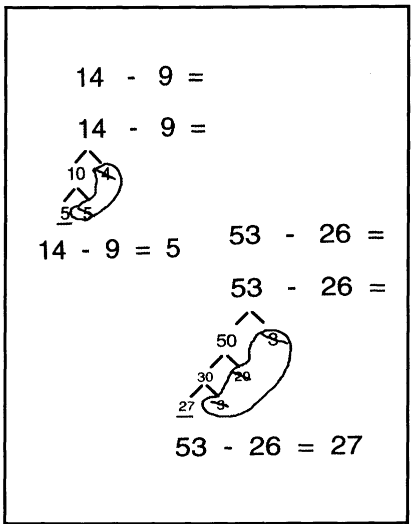


Figure 5
Method for Recording
Subtraction using Part/Whole
Strategies for "14 - 9"
and "53 - 26"

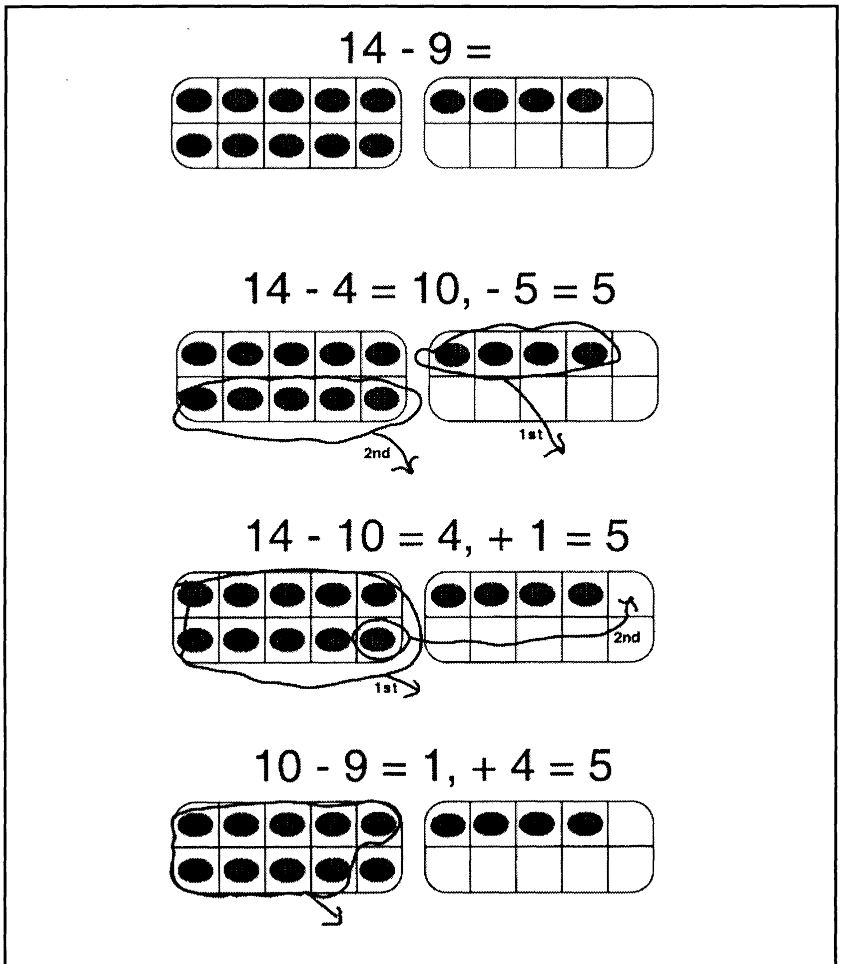


Figure 4
Several possible part-whole
strategies which could be used to
solve 14 - 9"

the two digits in 17 and the tens frames in their final state. For some children, this is the point at which they realise that the "1" in 17 means one group of ten.

Subtraction

A solid understanding of the additive composition of numbers makes the process of subtraction considerably easier to understand. Once again, the focus is on splitting the numbers in such a way that subtraction can be done without the need to count. Figure 4 shows the use of tens frames to model the ingenious methods used by Year 6 children to subtract 9 from 14 (see Young-Loveridge, 2001). The first method involves splitting the 9 into 4 and 5, subtracting the 4 from 14 to leave 10, and then taking the 5 from 10 to leave 5. The second method involves subtracting 10 (instead of 9) from 14 to leave 4, then putting one back with the 4 to make 5. The third method involves subtracting 9 from 10 to leave 1, then joining the 4 with the 1 to make 5. The advantage of having several equally good strategies is that it gives children the clear message that there is more than one acceptable way to work out the answer, and encourages them to think about the quantities themselves rather than executing mindless procedures such as counting or doing the written algorithm. In a British study of effective teachers of numeracy, this kind of "connectedness" was associated with better numeracy learning for pupils (Askew, 1999; Brown, 2000). Celebrating multiple solutions is something which I believe we should encourage far more.

Other Ways to Encourage the Use of Part/Whole Strategies

Tens frames provide just one way of helping children learn to use part/whole strategies. There are many other useful resources (see Young-Loveridge, 1999a, 1999b). One of my other favourites is bead-strings – a 20-bead bead-strings where two colours are alternated every five beads, and a 100-bead bead-strings where the two colours are alternated every ten beads. Bead-strings provide an excellent way to introduce the "Empty Number Line" described by Ken Carr and others (Beishuizen, 1999; Carr, 1998; Carr & Treffers, 1996). Another good resource is "play" money. I use only $1 coins plus $10 and $100 notes, in order to help children appreciate the equivalence of ten $1 coins to one $10 note, or ten $10 notes to one $100 note. Copy masters for the $10 and $100 notes can be photocopied onto coloured paper and cut up (see Ministry of Education, 2001c). The $1 coins can be purchased separately. It is likely that different materials appeal to different children. Whatever resources we choose to use, our goal should be to help children build an understanding of part/whole relationships among numbers and ways to use this understanding to solve number problems.

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