Helping Pre-service teachers to understand just why learning to count is not easy for young children

Rod Bramald

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Introduction

Learning to count is neither a simple nor a quickly acquired skill. It is complex and takes place over a period of time. This has been documented by many writers - see for example the work of Young-Loveridge (1999), Resnick (1983), Fuson et al (1982) or Ginsberg (1983). There are several sub-skills involved and total mastery of counting is generally not achieved in a sudden or short period of time although many teachers of young children will know of particular children who have made rapid changes in their perceived achievement levels in counting. Deciding whether or not a child can be said to be able to count is also not a simple task and almost bound to be the subject of judgements that may not attract universal agreement from the people most closely involved with the particular child. Some may think that the child can indeed count but was not performing at their best on the day the judgement was made and so on.

Likewise, learning to teach children to count is also a complex and time-consuming process. Put this task alongside the need to learn about children's acquisition of number concepts whilst at the same time trying to put the theory into practice demands a lot of pre-service teachers, especially in their first years of training.

> *"The research on pre-schoolers' mathematical competence shows that new-entrant teachers need to be aware of the rich informal knowledge of mathematics which children bring with them to school. Teachers can then organise the early mathematics curriculum so as to capitalise on that knowledge."*
>
> Young-Loveridge, 1987, p163

The problem for teacher trainers then is how best to help these new-entrant teachers to first see and then explore the children's existing knowledge.

The Context

As a teacher trainer from the UK, I recently arranged an exchange with a colleague doing a similar job in New Zealand. As part of my new role, I was teaching several groups of first year, undergraduate pre-service teachers prior to their going into NZ schools. The focus of their first school mathematics lesson was to be a one-to-one experience with a child of around five years of age who had only recently started school. Their challenge was to help the child in their learning about counting. My job was to prepare them to make the most of their time with the child. I was well acquainted with Freudenthal's dictum that "If you want to teach anyone anything in mathematics, first find out what they know" and so the pre-service teachers were being guided towards preparing for an exploratory interview which would enable them to do this. We needed to recognise the issues surrounding a young child's acquisition of the skill of counting.

The Lesson

We went through Gelman and Gallistel's five principles of counting:

- the one-one principle;
- the stable order principle;

- the cardinal principle;
- the abstraction principle; and
- the order irrelevance principle;

Gelman & Gallistel (1978) pp77-82

Illustrating each as we went. All seemed to be going well until we tried to explore what these meant from an adult point of view.

The students were given a suggested interview schedule to follow which included questions such as, "What number comes after 5?" and "What comes just before 7?". It was here that the first alarm bells began ringing for I recognised that most of the students seemed to think that these would be so easy that it was almost not worth asking. However, perhaps the question that said, "Suppose you have four beans and then I give I give you 3 more beans. How many beans do you now have?" would be worth asking.

It struck me that the students did not really appreciate just how difficult it is for young children to come to terms with all that counting involved and so I decided to put them into as near a similar position as I could. Comments from several new colleagues suggest that the activity would be worth sharing with more people.

Giving them new number names in a stable order

Learning new number names in a stable order is neither a quick nor easily mastered skill and I really wanted to try and by-pass this particular stage. If I was to do this, I needed them to use the new names in stable order that they already knew and could use fluently but one that wasn't the usual "one, two, three ...". We used the tonic sol-fa words from music so the new names in order were:

Doh re me fah sol lah tee

and we would use these for counting. Immediately, there were protests, "But there are not ten number words!" which was true so I explained that ten was always an arbitrary number that happened to match the number of our fingers. It is highly unlikely that had we evolved from octopuses or spiders that counting would have been in tens – it would almost certainly have been in eights. Had we evolved from a world where three toed sloths had been the dominant creature then base six.

Everyone was quickly able to repeat the words in the correct order and to demonstrate their one-one correspondance skills with objects up to tee. We could also agree on cardinality by saying that the word we ended with was the size of the set. For example, each of our hands has sol fingers. But what happens after tee?

Extending beyond single number words.

To make sense of this, we began by considering at our traditional decimal numbers looking first at the numerals. After 9 is 10 which uses the first number symbol plus a place holder which we call zero. After this we have the first symbol plus each of the original number symbols in order. Everyone could see the logic. But what about the words because this after all was what we were most interested in.

Ten is followed by some fairly peculiar words for the numbers which come between this and twenty. We decided to look a little further on than ten. What happens to numbers of higher values? As we come to a decade, we seem to add the suffix 'ty' as in six**ty**, seven**ty**, eigh**ty** and nine**ty**. Looking a little closer, we can see that we do it also for 'fourty' although we spell it slightly differently as forty and again for 'fivety' which we spell as fifty. Why doesn't this work for the early numbers? Logically shouldn't we have one-ty, two-ty and three-ty? The answer of course is yes but the English language is not totally logical and some rather idiosyncratic number names have evolved.

We agreed to ignore these peculiar words and decided to construct a totally logical number system by simply adopting the familiar suffix of 'ty' to indicate a new set of numbers. We could not call them decades since they are not sets of ten but at least we could now count beyond single word numbers.

Doh re me fah sol lah tee dohty

But now what? Looking back at our decimal numbers which take the new 'ty' suffix, we could see that each is followed by repeating the new 'ty' word plus each of the original single words in the same order:

Sixty (on its own, then) sixty-one sixty-two sixty-three sixty-four etc.

until we get to sixty-nine, then add 'ty' to the next single word and repeat the process.

Seventy seventy-one seventy-two etc.

Applying this to our new number names and stable order, counting becomes reasonably straight forward:

doh	re	me	fa	sol	lah
tee	dohty				
dohty-doh	dohty-re	dohty-me	dohty-fa	dohty-sol	
dohty-lah	dohty-tee	rety			

etc.

And with just a little more effort, we were able to go on further and construct the equivalent of our well known hundred square. However, and very importantly, we knew that the children would be working purely orally so we did not use a written version.

The group was now becoming quite proficient at counting up in this peculiar system but there were more protests about it being unnecessarily difficult. I pointed out that we are fortunate to be working in English even though it may have some peculiar words for numbers between 11 and 20. We should spare a thought for the French whose system is even more peculiar than ours! They too share our penchant for idiosyncratic words for the numbers in the teens but then appear to abandon logic totally when it comes to words for their decades. How does vingt relate to deux? Perhaps we can see something between trente and trois but saints preserve us from having to use four twenties (quatre-vingt) instead of eighty and why on earth do the nineties have to be four twenties plus the teen numbers again as in quatre vingt treize for 93?

One of the students was originally Austrian and so grew up learning German as her mother tongue. She commented subsequently,

> *"I found that it was a bit like learning to speak another language, it was new and I was unfamiliar with it. And even though I could relate it to my past experience of learning to count in English (my mother tongue is German) I was still struggling to make sense of it and say it in the right order."*
>
> Female, late teens

The next little hurdle was that of rather immature humour. The reader has no doubt spotted that coming up as our next 'ty' number is 'fahty' which sounds rather rude and of course its partner in the humour, 'fahty me'. This we confronted head on, joined in the general laughter then simply move on. (If you plan on following this exercise, my advice is the do the same: introduce it before they discover it for themselves! My wife is a teacher of modern foreign languages and she assures me that learning the German for father for the first time has similar effects.)

Moving on

By now, we could say the words of doh-re-me up to some really big numbers. So what? Could we say that we can count? We returned to the interview schedule and tried a few of the suggested questions for 5 year olds but substituted the new words and sounds. "What comes immediately after fah?" A few, mostly those with musical backgrounds, could bring sol quickly to mind. For the rest, it was back to the new number names and all around the classroom we could see and hear people repeating the order, "doh, re, me, fah, sol ..." as they studiously tapped or uncurled one finger at a time. Next, "What comes just before tee?" Again the same process of tagging individual fingers.

What about the Cardinality principle?

We counted sets of fingers on one hand (sol); and on both hands (dohty-re). We started asking questions around the room such as, "How many people at this table?" or "How many panes of glass in this window?" until finally, "How many people in this classroom?" (26). Almost everyone had to stand and point to each person one at a time whilst chanting the number sequence – a very explicit example of the one-one principle. However, there was little agreement on a single answer with several different answers being offered. Why? We tried to unpick the problem by counting altogether, slowly and out loud. It transpired that many had not remembered that the 'decade' numbers such as sixty, seventy, etc. or in doh-re-me, dohty, rety, etc. needed to be said on their own before starting to add the single words again. In base ten we have to say twenty before we can start again using twenty one, twenty two, ... etc. Do children do the same? Eventually we agreed on mety-re and, since we had already agreed that the last number we said was the size of the set, we had confirmed our cardinality principle again.

What about arithmetic?

Trying questions of arithmetic was quite difficult. "Suppose you have fah beans and then I give you me more beans. How many beans do you now have?" Oh dear – chaos reigned! Almost every single student reverted to using fingers and, it seemed, had to do the counting out loud! First on one hand, "Doh, re, me, fah". This was retained on one hand with the appropriate number of fingers (4) standing up. Then the other hand, "Doh, re, me" and they had three fingers standing. They knew instantly that it was 7 in traditional decimal counting but the answer needed to be in doh-re-me numbers so what did they do? They reverted to the count-all strategy and even though it was not necessary, almost every single person re-set their fingers to show 7 as 5 + 2 and then returned to the stable order, one-one and cardinality principles. "Doh, re, me, fah, sol, lah, tee" Choruses of "Tee" rang out followed almost immediately by peels of laughter as they watched each other trying to do it. We repeated this with similar questions such as, "What is sol more than rety-fah?" and after a little while, they were all convinced that they could now count in doh-re-me.

At this point, I threw in a googly (a cricketing term for a ball that spins the opposite way to normal). "What is dohty more than sol?" The vast majority began the same process of finger counting, converting to decimal numbers, finding the answer then translating back into doh-re-me to eventually get the right answer. I tried another version of the same question, "What is dohty more than rety-sol?" This was seen as getting harder except that a couple of students were able to get the answer very quickly. "It's mety-sol" They were assailed by their peers who wanted to know how they did it so quickly. "It's easy – it's like adding ten isn't it! 10 more than 17 is 27 and 10 more than 34 is 44. Dohty is like our 10 and so you just change the 'ty' word to the next one!" Daylight dawned and we had several examples where they all wanted to confirm their newly acquired skill of being able to add dohty to any number.

By now, everyone was ready to move on. We could do that and we could see why children find it hard.

But what about our interview questions? Didn't we also ask them to count backwards from 5 or 10 or even from 17? Can our doh-re-me counters do the same? Counting back from dohty was easy for the musicians again but not so easy for the rest. And what about from rety?

The next interview question was, "Can you count in twos for me?" Expected answer, "2, 4, 6, 8, ..." This was thought to be really easy until they tried it in doh-re-me. Everyone was back to fingers and saying the whole sequence very quietly to themselves like a mantra but saying every other number name out loud so that it *appeared as* if they could count in res.

Adding two numbers

On the blackboard, I put some of the initial strategies that children use when adding two single digit numbers such as 5 + 3:

- count- all;
- count on from the first; and
- count on from the larger.

In case these are not familiar to the reader, I have outlined them below.

Count-all: first count out each of the numbers, usually with concrete materials such blocks or toys or fingers. Next combine them into a single coherent group then count them all. For example, asked to add 3 and 4, the child counts out 3 fingers, then 4 fingers and finally counts all of them to get 7.

Count-on from the first: the child who can do this now knows about the cardinality principle and recognises that the first set ends at that count number and so simply continues the sequence from that point. For example, asked to add 3 and 5, the child starts with the 3 and, usually holding up a set of 5 fingers before continuing the count and says, "four, five, six, seven, eight" taking care to keep the one-one principle in mind as she counts on to eight.

And finally,

Count-on from the larger: This is a more sophisticated version of the previous strategy. The child recognises the efficiency of starting with the larger number first regardless of whether it is given as the first or second number. So 3 + 5 becomes something along the lines of: starts with the 5 in her head then holds up three fingers and says, "six, seven, eight". For a fuller and more in-depth look at this issue, see Thompson 1999.

Knowing which is larger is not something we can take for granted and I needed my students to appreciate this. I threw very quickly at a student, "Which is larger fah or sol?" Most were unable to answer this without first saying the names in sequence until they reached one of the given numbers. They tried to test each other with similar questions and my point was made very graphically.

After the school visit.

Once we were all back in the university after their first interview lesson with their particular five-year-old, I asked them to write down how it had gone. Many, though I must admit not all, were ready to say how much more meaningful they found their child's answers. They said things such as:

"During an exercise in class where "Doh Reh Me..." was used to replace 1,2,3... I began to understand how difficult it is for a young child to fully understand and use and name numbers. Before this experience I hadn't given much thought to the subject and had assumed that children picked-up numbers and naming numbers easily through every day life."

(female, early twenties)

> *"After (learning) Doh Reh Me ... I quickly realised that learning how to count numbers is rather difficult."*

(male, late teens)

> *"The exercises with counting in Doh Reh Me were helpful as an indication of how hard it is for the child to learn to count. I found it particularly hard to count in this way, as I had to constantly convert the Doh Reh Me's to numbers to follow their sequence."*

(female, early twenties)

And for me, the most revealing of these answers ....

> *"I have learnt that I must have patience and give adequate waiting time for an answer, instead of jumping in and trying to help the child when they are still thinking."*

(female, late teens)

Data results and interpretations

The students were asked to complete a questionnaire related to their experiences. An anlaysis of their responses follows:

Table 1: "How useful did you find learning in to count in doh-re-me?

	Very Useful	Useful	Neutral	Not much use	A real waste of time
n =	22	22	5		1
%	44	44	10		
		2			

They were asked to give a reason for their answer. To further aid the reader, the responses have been categorised according to the students' original views about the usefulness of the exercise. Thus, those students who thought the exercise was 'very useful' are in the 'VU' column and so on. The abbreviations used are:

VU students who originally thought the exercise was 'very useful'
U 'useful'

N	'neutral'
NVU	'Not very useful'
WoT	'Waste of time'

Table 2: "Can you say why?"

	Very useful	Useful	Neutral	Not much use	A real waste of time
No. of trainees in category	**22**	**22**	**5**		**1**
allows trainee to relate to how	15	17	1	(14)	
how they think a child would feel	(48)	(49)			
mentions that it was 'hard'	7	14	1		
or 'difficult'	(23)	(40)	(14)		
mentions personal memories of	4	3			
learning to count	(13)	(9)			
mentions it was complex or not	2	1			
natural	(6)	(3)			
'Enjoyed it' or other similar	1		1		
positive response	(3)		(14)		
negative			1		1
response			(14)		(100)
other responses	2		3		
(not differentiated)	(6)		(43)		
totals	31*	35*	7*		1
(% for column)	(99)	(99)	(99)		(100)

(fig. in brackets: %age of column responses)
* does not equal number of trainees since some gave multiple answers and others gave no answers

When they were asked if they could recognise any of the counting principles in their interview with the 5 year old child, perhaps the most interesting thing to emerge was not their naming of one of the Gelman-Gallistel principles but the fact most (48%) either couldn't give one or described something different such as 'Couldn't say what was just before 5' or '.. she was unable to bridge past 10'. This may indicate that the pre-service teachers themselves had not yet got a firm grasp of the principles and how to recognise them. The details are in Table 3 below.

Table 3: You were given the 5 five principles of counting as: *stable order; 1-1 correspondence; cardinality; abstarction* and *order irrelevence* Were you able to recognise any of these in your the child you interviewed?

	Very useful	Useful	Neutral	Not much use	A real waste of time
No. of trainees in this category	**22**	**22**	**5**		**1**
Stable order	6	4	1		
	(24)	(15)	(20)		
1-1 correspondence	4	4			
	(16)	(15)			
cardinality	1	1			
	(4)	(4)			
abstraction					
order relevance	2	5	1		
	(8)	(19)	(20)		
No answer or gave other	12	13	3		1
characteristic(s) not defined	(48)	(48)	(60)		(100)
totals	25*	27*	5*		1*
	(100)	(101)	(100)		(100)

* does not equal number of trainees since some gave multiple answers and some gave none

As a follow-up question, they were asked to say what they found easiest and hardest about their experience of both learning to count themselves (in doh-re-me) and their interpreting of what their 5 year old interviewee said in answer to their questions. Fewer than half of the sample responded to this question and their answers (see Table 4 below) are not particularly surprising but they do show a fairly even spread across their choice of hardest thing and a reasonably substantial agreement on what they found easiest.

Table 4: When you were learning doh-re-me counting, what was the hardest and/or easiest thing about it and why?

		%age
Hardest		
Adding and/or subtracting	7	30
Counting into 2-digit numbers	6	26
Remembering the stable order	6	26
Converting from Doh-re-me into decimal numbers	3	13
Counting backwards	1	4
Easiest		
Counting in single digits	10	59
Maintaining stable order	3	18
Othe answers (all single responses)	4	24

The students were then asked the same questions as in their schedule for interviewing a 5 year old except that the questions had been translated to Doh-re-me. *(the original, decimal questions are in brackets and italics).* In the table, the correct answer is marked with an asterisk* and brief comments for each particular question and set of responses are added at the end of that section.

Table 5: The 5 year old's questions but translated into doh-re-me

		Very useful	Useful	Neutral	Not much use	A real waste of time
4a What comes after fah?	Soh*	21	20	4		
1						
	Lah	1				
	? or no answer		2	1		

With so few errors there was no apparent common trait.

		Very useful	Useful	Neutral	Not much use	A real waste of time
4b What comes before mety fah? (What comes before 28)	mety-me*	18	15	1		1
	me					
	mety-soh	2	6	2		
	? or no answer	1	1	2		

The most common error here was to give mety-soh as the answer. This is the number immediately AFTER mety-fah suggesting that it is probably a mis-reading of the question.

		Very useful	Useful	Neutral	Not much use	A real waste of time
4c You have sol eans and I give you tee more. How many beans do you have? (You have 5 beans and I give you seven more. Howmany beans do you have?)	dohty-fah*	11	10			1
	dohty-sol	6	5	2		
	dohty-re		3			
	dohty-doh	1				
	7,11 or 12	1	1	1		
	mety-re	1				
	? or no answer	2	3	2		

The most common error here appears to be the omission of the decade number, dohty, resultng in an answer that is one number too large.

		Very useful	Useful	Neutral	Not much use	A real waste of time
4d Add together rety-me and dohty. (Add together 19 and 8[1])	mety-me*	6	7	2		
	27					
	mety-do		2			
	mety-tee		1			
	mety-re	2	3			
	9	1				
	mety-lah		1			
	mety		1			
	fahty-re					1
	? or no answer	9	9	3		

There was no real coherence to the incorrect answers here. I am unable to offer a plausile explanation for most but one of them, mety-re, is most probably explained by missing out the decade number dohty.

[1] The original question asked the child to add 10 to 19 which would result in a change only in the tens place. In order to test the same principle in Doh-re-me (base 8), the question was therefore changed to accommodate this by asking them to add dohty (8).

		Very useful	Useful	Neutral	Not much use	A real waste of time
4e What is mety-tee + fahty-fah? (What is 31 + 36)	dohty-dohty-me*	1				
	No answer	16	17	4		
	alternative incorrect answer	5	5	1		1

This proved to be step too far for most students as only an minority (26%) offered any sort of response and only 1 (2%) got the answer correct. Two digit addition before they were comfortably confident with one digit work follows much previous research findings about young children moved too quickly onto two digit calculations. The answer given here as dohty-dohty-me ought probably to have a new name for the third place digit to match the decimal 'hundred'. Since this was never discussed or suggested by any of the students, the addition of another dohty was accepted as correct.

		Very useful	Useful	Neutral	Not much use	A real waste of time
4f. What is tee fewer than dohty re? (What is 7 fewer than 12?)	me*	3	2	1		
	other incorrect answers	2	2	1		
	?or no answer	17	18	3		1

Although not particularly challenging, very few attempted this. It was towards the end of the questionnaire and fatigue may well have set in.

		Very useful	Useful	Neutral	Not much use	A real waste of time
4g Count backwards in your head from rety. Were you able to do it? (Count backwards from 16)	yes	1	1			
	No	15	13	5		
	? or not answer	6	8			

Only a tiny minority reported that they were able to count backwards. The vast majority reported that they couldn't do it.

"...our bright eyed and enthusiastic entrants to the profession are desperate to learn the essential skills so that they can get into classrooms and can start interacting with young children. We not only need to give them the tools with which to do this effectively but also the understanding of what goes on when the young are coming to grips with a whole new world of school in general and mathematics in particular."

Making sense of the experience

The findings of this small experiment are neither earth shattering nor particularly scientifically based. They are intended as an illustration to colleagues who are involved in training pre-service teachers. For these particular students, the experience appears to have been educative and useful. Their responses were genuinely honest I believe and the exercise may well be worthy of consideration by others.

Teacher training is already under the spotlight in many countries and no doubt will increasingly be so in the future. This will almost certainly mean that whatever experiences we include in our programmes will also be monitored and scrutinised for superficial relevance and practicality. I believe this particular activity and all those similar to it such as the Alphabetland exercise in England's National Numeracy Strategy training materials (DfEE 1999) will enhance our credibility with the public. I have shown this to several people who are neither teachers nor teacher trainers and they have all commented upon it positively too.

In my experience, our bright eyed and enthusiastic entrants to the profession are desperate to learn the essential skills so that they can get into classrooms and can start interacting with young children. We not only need to give them the tools with which to do this effectively but also the understanding of what goes on when the young are coming to grips with a whole new world of school in general and mathematics in particular.

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