Young children and geometry

Ken Carr and Carol Carr

"As with many other strands in the mathematics curriculum, geometry is now thought of as possessing content that may be introduced to students at a much earlier age than was once thought..."

The parallel here is with number, of course, where research going back several years now (e.g. Carpenter and Moser, 1982) showed that if number problems were presented to children in a manner that made sense to them then they would be able to understand and solve problems that the 'wisdom' .of the time suggested might be beyond them.

Geometry (sometimes referred to as 'Shape and Space' in mathematics education) is seen as an integral part of the mathematics curriculum in most countries in the world. Our own curriculum document (Ministry of Education, 1992) lists it as one of six learning strands. The NCTM's (US) standards (NCTM, 1989) has geometry as one of nine content areas. Several other content strands in mathematics have explicit connections to geometry. Algebra may include patterns in geometry, aspects of probability link with geometry, and connections are commonly made to measurement. Collier and Pateracki (1999, p. 412), when discussing geometry in the middle school assert that "no longer is it to be a 'frills' topic relegated to the back of commonly used textbooks nor restricted to identifying figures and reciting a litany of their properties. Instead, geometry should encompass a variety of experiences in which middle schoolers create their own understanding of the relationships to our multidimensional world". The same could be said for younger children.

Children learn geometrical concepts long before they come to school. As they explore the natural and made environment they sort and distinguish between shapes, build with shapes, and discover how they may (or may not) fit together. They discover the properties of some shapes (Bartels, 1998). As well they may informally investigate transformation geometry as they slide, rotate and flip blocks, tiles and other shapes. All of these activities may be done in a problem solving context that has legitimacy for the child herself/himself. Play is a common part of a child's existence, and whether the play involves fantasy elements or not, geometry may feature as an integral part of what the child is doing.

One of the problems that teachers of geometry have faced has been the relative dearth if resources available to help plan units of work. Although the use of texts may obviate this to some extent in the middle and high school, this in turn can lead to another problem - a very formal approach to teaching a strand that has inherent interest and

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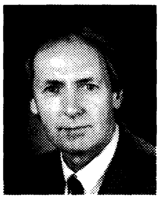
applications to the world outside the walls of the classroom (it is interesting to note here that the word 'geometry' itself comes from the Greek *geometrein* meaning 'to measure the earth'). Teachers of younger students can turn to the curriculum document (Ministry of Education, 1992) for overall guidance, but there is insufficient content and detail here to give much help. A curriculum document cannot be expected to carry sufficient content for the detailed planning that is required for units of work. Thus teachers are left to gather ideas, resources, and plans from their peers and from teacher journals and other commercial publications - fortunately we have a history of utilizing scarce materials and sharing these with others in the profession (Openshaw, 1995).

Mathematics educationalists



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(e.g. Burns, 1992) agree that spatial ability has many applications in the world outside school, "a fact that any adult encounters when having to figure quantities for wallpaper, floor covering, paint, fabric, lawn needs, or a myriad of home projects. Geometric concepts and relationships are also essential to many branches of industry, the building trades, interior design, architecture, as well as other work situations" (Burns, 1992, p. 80). Thus it may be relatively easy for teachers and students to find contexts that are worth exploring in geometry. And especially those that are realistic to the student.

A motivation for this teaching experiment was to see whether or not a group of five and six year olds could successfully complete some activities in geometry that might not ordinarily be given to students in the first years of schooling. The intention, however, was not to try to accelerate these students through the curriculum, but rather give them experiences that might enrich them laterally.

The activities *per se* came from suggestions in the curriculum document (Ministry of Education, 1992), and from other sources (e.g. Woodham, 1987). Some of these activities had been used with children by student teachers from the University of Waikato in their mathematics education courses, and had been seen to help children learn the relevant geometrical concepts. However in these episodes the children had been generally in years four to six.

It is interesting to note that in the recent TIMSS study (Ministry of Education, 1997) geometry was one of the content areas in which New Zealand students (standard two) performed at a level (58.1) just above the international mean percent (55.6). Gender differences were apparent in the geometry results as well - "...standard two girls scored significantly higher than their male peers on four content areas, with the biggest differences in Geometry and Patterns..." (Garden, 1997, p. 80).

The 'easy' items (in TIMMS) were defined as those where at least 80% of the students answered a question correctly. The strand containing the most number of 'easy' items was geometry. These items asked students to use their *knowing* skills as well as more complex procedures. For example item J1 illustrated a hexagon divided into six triangles and asked students to name these shapes. Item L3 required students to name the object in a board game using given coordinates. The report notes that "the relatively higher performance of standard 2 and standard 3 students on geometry items reflects the greater emphasis given to geometry at the junior school level in New Zealand than in many countries" (Garden, 1997, p. 86).

The achievement aims for geometry for New Zealand students are to:

- gain a knowledge of geometrical relations in two and three dimensions, and recognise and appreciate their occurrence in the environment;
- develop spatial awareness and the ability to recognise and make use of the geometrical properties and symmetries of everyday objects;
- develop the ability to use geometrical models as aids to solving practical problems in time and space.

The achievement aims for the current teaching experiment came from level 1, Ministry of Education (1992). They included:

- identify and describe in their own language 2-dimensional shapes.
- classify objects by shape attributes.
- follow and give a sequence of instructions related to movement and position.
- create and talk about repeating patterns

What Was Done

The children , from a normal school attached to the University of Waikato in Hamilton, were all aged five or six. Most had been at school for about a year. They were described (by their teachers) as having some aptitude for and interest in mathematics that the teachers had noticed. The children (six girls, four boys) came from five different classrooms.

The five lessons in the series were spread over two weeks. Each lesson took place in a small classroom that is often used by university students who work in the school. The lessons started with the group working together as the problems were introduced and discussed, then independent study took place as the students worked on their particular solutions. However as they worked with their response they were seated near each other and shared ideas and strategies as is normal in a mathematics class of today. At the conclusion of each session the students talked about what they had done, and responded to questions (usually of clarification) that the authors and their peers might ask.

The teaching unit that was used is attached as Appendix A.

Findings

The major findings were as follows.

1. The children were very motivated by and interested in the activities. They remained on task for the duration of each session, and were sufficiently challenged by what they did.
2. Interaction between the children was at a high level during each session. They were keen to show each other their work, and continually chatted among each other.
3. The mathematical objectives for the unit appeared to be met. Perhaps the naming of the geometric shapes was the easiest task for the children - they had a good knowledge of shape names right from the outset, and had obviously been exposed to working with shapes. The only shapes they were unsure of were hexagon and trapezium, but these were soon learned. Young children seem to possess an almost 'natural' ability to

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quickly learn and remember the names of the two-dimensional shapes. And (with a little guidance) to notice their occurrence in the natural and made environments.

4. The teaching style used in this unit was based on at least two theoretical models. The planning was quite precise, and instructional objectives were generated for each phase of the unit. Certainly the children themselves had no say in the content of each lesson.

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This was the traditional form of planning that contains elements of quasi-behaviourism some might claim. On the other hand, during the actual teaching and learning of the sessions the teachers tried to adopt a 'social constructivist' style of teaching whereby they;

- listened to the questions of the children as they worked and attempted to guide the children to a satisfactory solution (or partial solution) to their question.
- did not give explicit step-by-step instructions at any stage.
- built in a process where-by the children shared their findings with their peers.
- encouraged a form of 'negotiation of meaning' as the children worked through their

activities. This was achieved by asking the children to reflect on what they were doing, suggesting alternative ways of looking at a problem, and making sure that the children were aware of what others were doing and how others might have solved a problem similar to the one they were at that time facing.
- asking very open-ended questions that did not provide answers but guided the child back to solving the problem for herself/himself. Teacher questions of this type included 'I wonder how we might...?', 'I wonder what would happen if we...?', 'Is there another way we could...?', etc.

5. Assessment of learning was by observing the children as they worked, and by interviewing each of them at the conclusion of the unit. Samples of work were kept to go into the ongoing work portfolios that the

schools keeps for all its students. Interview results showed that most children had attained the unit goals.

6. One activity in the unit introduced children to an aspect of geometry that may be used with older pupils. This activity, tessellating fish (Woodham, 1987) proved to be very popular. The children understood the concept of a tessellating pattern, and were able to produce simple tessellating designs. The important point was that tessellations were placed in realistic contexts that the children could relate to. With the soft rubber mosaic shapes the children designed a repeating pattern for a bathroom floor. First they used one shape, and through this discovered that not all two dimensional shapes will tessellate. Next they used two different shapes, and again

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foumd that not all combinations of two shapes will tessellate. The fish design comes from a rectangle that is modified according to some of the principles that Escher used in his work. Again it was a context that the children could relate to.

Discussion

This teaching experiment suggested to the researchers that::

- children in the early years of schooling are capable of learning quite 'advanced' concepts in geometry if these are embedded in contexts that make sense to them. These contexts should come from the experiences of the child so that a useful bridge is made between the reality of the child and the mathematical concepts being introduced. We *"mathematics is an inherently interesting and fascinating subject for learners."*

were reminded here of Hughes' work many years ago in early number, where he introduced activities that helped children 'bridge the gap' (as he called it) between the symbols that the child is familiar with to the expressions and symbols of early mathematics (Hughes, 1986).

- mathematics is an inherently interesting and fascinating subject for learners. We were aware of the high involvement of the children in the tasks, and the anticipation they showed at the start of each session. There were no management problems with these learners - which reminded us of the maxim that the first step a teacher should take to do with control and management is to ensure that the classroom programme is suiting the needs of the learners and engaging them in productive activities.
- observation of children as they solve problems was a powerful way of assessing learning. There was no need to 'set a test' at the conclusion of this unit to measure learning. We knew through our observations whether or not the children had met the objectives that the unit contained, and we could get a picture on their attitude, cooperation and perseverance. The work samples collected went into their portfolios, and would be used at the mid-year and end-of-year assessment points. This assessment could be seen as formative, but contributing towards a summation at two times in the school year. The pictures that the children constructed could also be used at parent-teacher times when they would provide a useful talking point and an illustration of new approaches for teaching and learning geometry.

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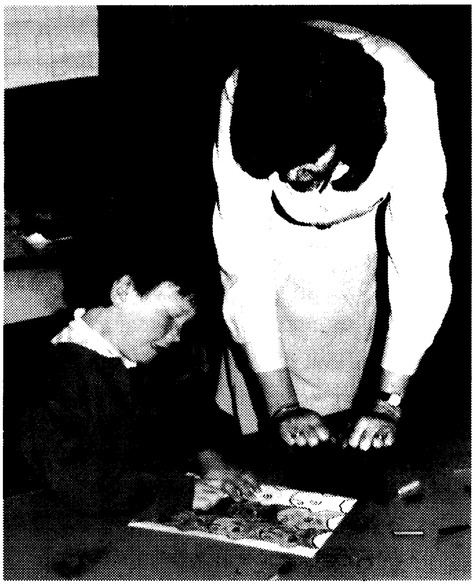
- some commonly-used activities (e.g. pentominoes, math games) can be easily modified to make them suitable for learners at different ages, as was done in the present study. Thus the store of useful contexts available to teachers may not be as limited as first thought.
- transformation geometry provided the basis for the activities. In other words the children were studying the effects of movements on shapes. Translation, rotation and reflection were performed intuitively by the children, and in contexts that were, for them, realistic. Mathematically, the important point is that the basis had been laid for further work in these areas in the years ahead.



- A teaching experiment such as this can give valuable insights into mathematics learning, or learning in any curriculum area. The intention with this series of lesson was not to 'prove' that one method of teaching was better than another, but to gather data that might inform our future practice.

Appendix A

A Unit in Geometry

(For Level One Children)

1. ***Content Strand/s***

   This unit focusses on the Geometry strand, but also touches on aspects from Number and Algebra.

2. ***Topics***

   This unit helps children to:
   - identify the common regular two-dimensional shapes.
   - explore simple pentominoes (with four squares).
   - use shapes to cover a surface without leaving spaces (tiling with mosaic shapes).
   - modify a regular polygon so that it will still tesssellate and will incorporate aspects of creative design (after M. C. Escher).

3. ***Content Objectives***

   To help the students to:
   - identify and describe in their own words two-dimensional shapes.
   - discuss and design symmetrical and asymmetrical patterns and look for examplesof these in the natural and made environment.
   - fit shapes together to cover a surface.
   - understand that not all shapes will tessellate.
   - design and use a tessellating shape to cover a surface.

4. ***Mathematical Process Objectives***

   To help the students to:
   - solve problems involving mosaic shapes.
   - discuss their ideas with their peers.
   - create, continue and describe a shape that will cover a surface.

5. ***Contexts***

   As stated above, the contexts will include problem solving situations where students:
   - manipulate and explore rubber mosaic shapes.
   - try to cover an area with one

and two shapes.

- design a pattern for a bathroom floor using shapes that tessellate.
- make an Escher-type design/ picture.

6. Assessment

Assessment will be through:

- Observation of the students as they work.
- self-assessment where the students reflect on what they have learned during the unit.
- analysis of work samples that the students produce e.g. can they produce symmetrical and repeating patterns.

7. Lesson Sequence

Day One:

1. The students will be introduced to a range of shapes through the mosaic blocks. Through sorting they will discover:
   - same and different
   - reasons why this may be the case.

   They will discuss their findings, and build up a chart of the regular polygons 'square', 'rectangle', 'hexagon', 'trapezium', 'diamond' (rhombus), 'circle' and 'semi-circle'. These names will be written alongside each shape. The students will be encouraged to discover the attributes of each shape. e.g. how many sides, corners etc.
2. A version of the pentominoes activity using four squares. The students are asked to make as many different shapes as they can from the squares using the normal qualifications for constructing pentomino shapes, i.e. joining complete sides without overlap.
   They are given several 2 1/2 cm by 2 1/2 cm squares of paper and some glue, and encouraged to put the squares together to make as many DIFFERENT shapes as possible. These are then stuck on to the accompanying sheet of paper (variations can be achieved by rotating and/or reflecting shapes). Each paper square must be placed edge-to-edge - no overlaps or staggered placement (i.e. not like a brick wall) .

Day Two:

Exploring patterns with mosaic shapes

1. Putting shapes together to cover a flat surface without leaving gaps ('like a jigsaw puzzle") A piece of A4 sized paper given to each child. Discussion of the students' findings and creations.
2. Choosing <u>one</u> shape and using this to cover a surface.
3. Choosing <u>two</u> shapes that can be used repeatedly to cover a surface. (This 'controls' the level of difficulty for the child).
4. Drawing this design and then colouring it. Some children may achieve a repeating pattern with their colouring.

Day Three:

Continuing to explore shapes that will tessellate

1. Students to work with squares, rectangles, and right-angle triangles of varying sizes.
2. Designing a pattern for their bathroom floor at home. Using pre-cut coloured paper shapes to create the designs. Limit this to a choice of two shapes. Some may produce a one-off design. Some may create regular repeated patterns.

Days Four and Five:

1. Tessellating Fish activity in the style of Escher. This activity involves the students in:
   - working with a rectangular shape
   - making a new shape from this that will also tessellate. Modifying the original shape by cutting out a simple shape on two sides and resiting them on opposite edges
   - making a stencil that can be traced around. repeating this shape onto a sheet of paper (perhaps A4 size).
   - creating an attractive design through appropriate colouring e.g. repeating patterns.

8. References

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9. Resources

Coloured paper, scissors, rubber mosaic shapes, light card, sellotape.

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