How big is the waka? Measurement with young children

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This paper explores the teaching of measurement with a class of five year olds. The children were read a story that was based on a fishing trip in a waka. Next day the students were set the task of measuring how 'big' the waka was - an outline of the canoe had been taped to the classroom floor. Over five sessions this open-ended problem was solved. The teacher facilitated discussion, elicited questions from the children, provided praise and encouragement, recognised all contributions from the children and followed the evolvement of the children's thinking and actions. Implications for teaching measurement using an alternative approach are discussed.

Introduction

Measurement is often thought of as providing relevant and meaningful mathematical contexts for students in the primary school. "Measurement clearly provides natural links between our experiential world and the world of numbers" (Pope, 1994, p.100). It is now an important part of the mathematics curriculum, and teachers attempt to provide a range contexts as students develop measurement concepts.

Much has been written about measurement in the mathematics classroom, and developmentalists (e.g. Piaget) have paid attention to the progression of learners as they develop measurement ideas. In general these researchers have pointed up the gradual refinement of skills and understandings through which students pass as they move to more sophisticated and complete conceptions. Piaget's conservation tasks, though now subject to critical review, were essentially tapping into measurement concepts in the areas of length, continuous quantity, discontinuous quantity, weight, area, etc.

Why teach measurement? Perhaps because of our human history. Since early times people have attempted to order, compare and inform themselves and others about the environment through measurements. From a mathematics education point of view measurement has provided a natural way of learning about numbers. And measurement applicability to many occupations (Milroy, 1992) has also helped ensure its important place in the mathematics curriculum.

Measurement is also a part of other strands of the mathematics curriculum. Burns (1992, p.46) notes that "measurement gives children practical applications for the computational skills they are learning. It also provides a way to tie basic geometric concepts to number concepts. In addition, measurement offers opportunities for

interdisciplinary learning in subjects such as social studies, geography, science, industrial arts, home economics, art and music".

Because of this usefulness measurement was one of the first mathematical processes developed by humans. It is not difficult to imagine contexts in which it was required as the human species evolved. As symbols were developed that could record data (an important step forward), the assigning of units of measurement, and what these stood for, became powerful information. The units and symbols themselves, however, were entirely arbitrary. As culture developed, systems were devised that best made sense for that particular culture.

An interesting aside here is that of the 'global village', and its impact on systems of standardised measurements. Cultural variations in the units and language of measurement have proven a difficulty in a world of international trade and communication. We all know of the continuing attempts to have a universal system of measurement and the movement towards this. Understandably, nations that have constructed and used a particular set of measurement conventions for many years are loathe to change - witness the U.S.' unwillingness to adopt the metric system.

Measurement in schools

In the classroom, learners may move gradually towards the use of standard units of measurement. Typically teachers provide activities that would help young learners realise the confusions resulting through using non-standard units of measurements (e.g. paces, handspans, cup-fullers, etc) to measure the same object, and gradually learners would come to see the need for standardised units so that sensible comparisons and communication might take place. Sidle (1996) describes how the children in her class made footprints to measure things in the classroom:

> *... One boy in the class had a new baby brother Jeremy. I asked Jeremy's mother for a copy of his footprint and made photocopies for the class to use. They also used my footprint ...and made tracings of their own feet .... The children reported the size of various things in the room in Jeremy feet, Sidle feet, or their own foot measure. These children are well on their way to realising the need to standardise "foot" as a unit of measure.* (Sidle, 1996, p.450)

Burns (1992), Pope (1994) and other mathematics educators describe many concepts in measurement. These include:

- making comparisons by putting things together and then perhaps ordering these objects
- using non-standard units to measure a variety of objects
- matching something to a scale comprising standard units
- being able to select and use appropriate units of measurement

- estimating before measuring
- understanding that the physical act of measuring is, at best, an approximation.

Many approaches to teaching measurement with young children provide quite intensive and extensive teacher guidance for many activities. This may be through the use of worksheets, other photocopied resources, carefully described lesson steps in a commercial publication, or units of work produced by teachers and occasionally advisers of mathematics. Often, in these resources, a teacher-designed problem is presented to the students, lesson steps are followed, and the teacher will carefully check and guide the students' thinking so that they produce the answer the teacher may have in mind all along. Here is an example of this:

> *Then the student responded, and the teacher continued with "Good, the ball would weigh two pounds'. Pointing to the left scale she said, 'If the ball weighs two pounds then the cylinder would weigh .... [the students responded]. Good, the cylinder would weigh one pound ...'* (Spungin, 1996, p.177)

Even the more adventurous publications tend to provide detailed help for teachers, in spite of claiming that they are not following a 'traditional approach' (e.g. Baker *et al,* 1992).

Although measurement has always had an important place in mathematics *per se*, in past years it was treated quite incidentally and perhaps less seriously in the classroom. It was often seen as a topic that could provide a number of realistic contexts for other aspects of mathematics such as number and number operations, statistics, and geometry. It was regarded, in one sense, as a peripheral part of the mathematics curriculum, and the treatment that it received in the primary mathematics classroom reflected this. Those days have gone.

The curriculum document *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992) identified measurement as one of six strands, or major conceptual areas, within the mathematics curriculum. This is a reflection of the importance, relevance, realism and potential for practical application that make up a rationale for measurement in today's mathematics curriculum. The achievement aims in the above document are to provide opportunities for students to:

- develop knowledge and understanding of systems of measurement and their use and interpretation
- develop confidence and competence in using instruments and measuring devices
- predict and calculate the effects of changes in variables and rate of change of variables on systems representable by simple mathematical models.

In the present study the objective/learning experience most closely comes from Level 1:

> *Students should be;*
>
> - *estimating and measuring with non-standard units to find length,*

area, mass and volume (Ministry of Education, 1992, p.59)

The present study

Schifter's (1996) research provided a stimulus. She described how a group of young children was challenged to work out the length of a large object. The teacher gave little guidance to the class, and waited for children to come up with strategies for measuring the length. Initially the students were unable to respond. Then over a period of several days the students tried various methods, eventually arriving at fruitful strategies and ways of recording the data. Schifter's study had pointed up the potential for young children to solve measurement problems when the teacher did not play the 'traditional' role of mentor and transmitter of information and heuristics.

We decided to leave the problem even more open-ended than Schifter's and also to incorporate the use of the children's question in the teaching and learning.
The research questions were:

- Could five year olds develop strategies to solve a measurement problem with only minimal teacher guidance?
- What use are children's questions in the learning process in mathematics?
- Could the five-year-old children share ideas and communicate with each other?
- Is it possible for a mathematics lesson to have credible mathematical outcomes when the teacher has minimal involvement in terms of guiding and shaping student learning?
- How might the students interpret the word 'big' in the context of this study?

What was done

The children were a class of 26 five-year-olds from a suburban school in Hamilton. The group was racially mixed with 50% pakeha, 20% Asian, 20% Maori, and 10% African students. There were equal numbers of boys and girls in the class.

Each morning in the time from 9.15 to 9.45 the researcher worked with the whole class. All sessions were audio-taped, and notes were kept of significant events during lessons. Planning for the sessions was minimal in the sense that the aims were kept in mind by the researchers but lesson plans with objectives and lesson steps were not used. Each session was discussed with the classroom teacher both before and after. Interesting student responses were noted, and possible approaches for the next session were planned.

The researcher was the main teacher for each session, but the classroom teacher also played a key role in observing the children, listening to their comments, and noting significant ideas. As well, the teacher worked alongside the students as they moved to a solution that satisfied them.

The key question was 'how big is the waka?'. The adjective 'big' was deliberately chosen because it allows different interpretations. This fitted well with the overall tenor of this study. It was the student's conceptions, constructions and processes that were being sought. The objectives-driven approach was not seen as appropriate for this study for many reasons but especially because the approach was not congruent with the research questions.

Results

On day One the children were read 'The Fish of Maui'. This story told of a fishing trip made by Maui and his brothers. During the reading of the story the waka (canoe) was noted where it appeared in the illustrations. No specific measurement questions or comments were explored.

Before the second session began a masking-tape outline of a waka was placed on the classroom floor. This outline measured approximately six metres long with a beam of one metre.

As the students arrived at school they immediately noticed the outline and inspected it. They did this by walking or running around it, sitting inside it, picking at the tape with their fingers, and involving themselves in fantasy play using the outline as a prop. For example, three children sat inside the outline and taking some coloured sticks from the mathematics table began to 'paddle'. This was a typical game, the outline being clearly recognised by most of the students as that of some type of boat. Just before school began a parent of a child in the classroom walked into the room, sat inside the 'waka', took hold of a student, and pretended to throw the child in the 'water' saying "I'll throw you in the water Henry!". This, of course, convinced students who were still unsure that the outline was indeed meant to be a canoe.

The second session allowed the students to comment on the shape, and to pose questions that they might like to explore. The questions were recorded on a whiteboard and discussed in turn. Some of the questions were discussed at this stage, but the children's attention indicated that enough had been done for the day.

Session three began by asking the students to look at the questions they had generated the previous day, and then to decide on the ones that they would like to investigate further. These latter questions were:

- How are we going to find out how big the waka is?
- How many children could fit inside it?'

Initially the students found it difficult to imagine how they might work out how 'big' the waka was. This was, perhaps, to be expected because this type of activity was new to these five-year-olds. As a previous measurement activity they had made jumps outside and compared their distance with that of their peers (in relation to a study of the Olympic Games). But trying to measure a longer object with little (or no) teacher

guidance was novel. The researcher waited for the students to suggest ideas, but very few were made. However, they were starting to think about the activity - the scene had been set.

Eventually they suggested several classroom objects that they might use for working out the length of the waka. These included using:

- tape rulers
- scissors
- sticks
- crayons
- blocks
- pencils
- long boards from the block bin
- cut out paper
- people standing on the tape
- people lying down
- a ruler

At this point the classroom teacher intervened and suggested that the children wander around the classroom and try to collect objects that might be useful in measuring how big the waka was. They walked around the classroom to find these. Individually or in groups they tried out their methods. It was during this time, of course, that counting skills were put to use. Some of the children could count no further than four or five, while others in the class up to one hundred. The students counted outloud as they attempted to measure the waka. Their peers listened, and practised their own counting skills.

Session three ended with some children using a chosen object to measure the waka. Claudia brought a bundle of yellow plastic sticks to the waka outline and laid these end-to-end along one side of the outline. Shardell joined in adding another six or so. Victor brought a bundle of crayons and put them end-to end next to the yellow sticks. Other children then joined in until one side of the waka had sticks along its length.

The researcher then asked the children if this could be used to work out how big the waka was. There was a pause then Kathy said, "Put them (the sticks) alongside and know how many it takes to go around ... and you know how many that it takes to go around". Kathy then counted the sticks - "Twenty! ... it's twenty and twenty on each side." Other children then volunteered to count the sticks. Kevin got fifteen. Greetje got twenty-five.

At this time the researcher asked the class if they could now say how big the waka was. One child responded, "Twenty-nine ... twenty-nine metres."

Session four began with the researcher asking the children to think about the

previous 'lesson' and then to think about other ways of measuring the waka. These ideas were recorded on the whiteboard as the children offered them. They included the use of blocks, pencils, people lying down, long boards, some cut-out paper, and people standing next to each other. These ideas were explored orally with the children, but the session was cut short because of an interruption to the classroom.

Session five started with a whole-class discussion. The ideas from the previous sessions were discussed one by one, and the children told the researcher which idea had been theirs. The class was then asked which of these ideas they would like to use to find out the waka's length. Howard decided he would like to measure the waka with pencils. The researcher asked if the pencils were different lengths "would that matter?" "No" replied the child.

During this session the researcher used the work "big" to describe the length of the waka. A child corrected him - "how long" he stressed. The children were now starting to use 'long' rather than 'big' - this had developed naturally out of the context being used.

The students wanted to try two ways of finding out the length of the waka - the first by having people stand on the outline, the second with children lying down. These methods were tried. It was found that twenty-two children could stand on one side - Cathy thought that the waka was twenty-two big because of this, but then thought it might also be twenty-two metres long. Henry and Corrie decided that the waka could be measured by people lying down:

> *People lying down ... they lie down on their backs so ... and some more children with them ... and some adults with them on that side (of the waka outline) ... they lie down and we count their tummies.*

These children then explained how people would lie feet-against-head in order for the measurement to be more accurate. Five children could fit lying along one side of the waka. Again in term 'metres' was used by the children - "five metres" said Corrie. However, five children did not quite fit on the other side. Ewen's legs were extending beyond the end to the tape. The children counted their peers again. Sally exclaimed, "five people lying down and half of someone else!" She then self-corrected - "Four and a half." This was explored with the whole class. At this point the session closed.

Next day seven students from the class were interviewed about the investigation. These conversations were taped in a corner of the classroom. The children were asked what they had learned from the study - this is discussed in the next section.

Discussion

Several teaching and learning points come out of this small study.

1. The children were able to meet the learning objective (Level 1) from the Ministry

of Education (1992). After a 'slow' start to the series of lessons, they became more interactive and willing to share their ideas. As this happened the measurement concepts were confronted and skills in this area developed. However, it is interesting to note that some students appeared to be ready to use metric units as well as non-standard measurements - a learning experience suggested for Level 2 students.

Estimation skills are an important part of measurement, and this study showed how these may be promoted when children are given a change to try out their ideas. As the students attempted to answer the questions that they themselves had set, then inevitably they were forced into estimating and checking. Perhaps because there were no teacher-set activities (e.g. worksheets) and no written recording of their estimations, the children felt more comfortable at 'having a try' (cf. Koloto, 1989).

2. Related to the above, one wonders if the aims/objectives for Level 1 students in measurement are, in fact, entirely appropriate. The experience of the researcher and class teachers suggests that standard units of measurements should be discussed, at least, informally, with five year olds. Their terms 'metre' and 'centimetre' occurred spontaneously in this study - when this happens teachers should be free to explore these concepts (at a suitable level) with their students. There were several chances for a teacher to explore a concept of 'metre' in this study.

3. The problem in this study was deliberately open-ended. This is, perhaps, in contrast to the usual approach, and to the learning experiences suggested in the curriculum document (Ministry of Education, 1992). For example, one suggested activity in this document is children measuring an object with different shoe sized, then comparing the counts. As it is presented this is very structured. But this same activity may be easily modified to a more open study which may have more purpose and realism (than simply lining up shoes on a given length) if it were embedded in some problem-solving context.

4. The study has planning implications for a unit of work in mathematics. Because of the nature of the task, planning prescribed lesson content and sequences was not possible. At the start of the unit (on the basis of Schifter's study) it was thought that there might be five sessions. The students' questions formed the basis for part of the lesson content - these could not be predicted. Thus the precise content for each lesson was not known, and any attempt at writing lesson and unit objectives was impossible. What was possible, though, was a

series of unit aims. These, along with teacher reflections and judgments at the conclusion of each session, were sufficient to guide the unit's development.
It is important to note, however, that a great deal of planning did go into this activity. The topic was researched by the writer and the classroom teacher. Suitable resources were made available to the children. The curriculum document was studied. As Cobb (1990, p.148) notes:

> *... this form of teaching requires far more from the teacher. Ideally, the teacher should have a deep, relational understanding of the subject and be knowledgeable about possible courses of conceptual development in specific areas of mathematics.*

5. Language affects all our learning. The words we use can often be interpreted in a variety of ways - this is particularly so in mathematics. In this study the generic word 'big' was used. This was deliberately chosen as we wanted to find out how the students would interpret it. In the event it came to be used as a synonym for 'long', as the students saw that they were investigating a problem about length. Is this suggesting that a context that has meaning for learners will help them refine their language?

6. The children asked a number of questions during session two. From the six or so that were recorded two major ones were selected (see above). The selection was done in conjunction with the children in response to the problem that was being posed. It was interesting for the researcher and the class teacher that children were able to construct suitable questions even though they had not done a great deal of this in the past in the teaching-learning cycle.

7. The interviews with seven children at the conclusion of the study asked them to describe the length of the waka. Most of them said it was four, or five, or four and a half people long. Lizzie claimed it was "five metres long ... he lied down on it ... five metres long because there was five people in the waka." Katherine remembered both the standing-up and lying-down length, " ... and when you're standing on the sellotape only a little part of you ... and when you're lying down there's all of you ... four and a half lying down." All students had formed ideas about the length of the waka.

8. What did the children learn? Assessment was based on observation (a very powerful form of assessment), recorded responses of the students, and interviews with a small but representative sample at the conclusion of the study. The data showed that these five year olds had learned to:

- ask questions that led to fruitful investigations

- use non-standard units to measure length
- develop their counting skills
- select and use appropriate units
- make estimations
- realise that the act of measuring is an approximation
- develop concepts about fractional numbers
- communicate their ideas to others
- try out a variety of strategies for solving the problem
- become more confident in using measurement instruments
- discuss and attempt to use standard units of measurement

9. This study suggests a way of approaching the topic of measurement with young children. The study was underpinned by a constructivist view of teaching and learning. Cobb (1990) has outlined some of the difficulties with proposing teaching approaches in this area:

> *The best that can be done is to propose general instructional heuristics compatible with teaching by negotiation and to suggest a variety of specific activities and interventions that might work with some children ... it is clear that the constructivist view of instruction implies that the teacher must be a reflective pedagogical problem solver who, in effect, conducts an informal research programme.* (Cobb, 1990, p.149)

This, essentially, was the approach of the current study.

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