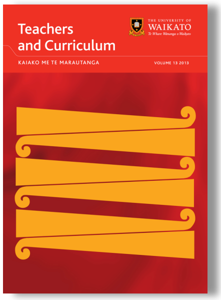

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MAKING MULTIPLICATION MEANINGFUL: TEACHING FOR CONCEPTUAL UNDERSTANDING

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Abstract

The term numeracy is used widely in schools today and brings with it the expectation that students will be taught both how to do the mathematics, alongside an understanding of the concepts associated with the procedural application. One issue, which has arisen with the terminology 'numeracy classroom', is how to best support teachers to enhance their teaching of mathematics to allow this understanding to occur. This article stems from a larger research study that analysed the professional knowledge of teachers when teaching numeracy, and the impact their mathematics knowledge and procedural application had on children's learning. This article presents observations of three teachers teaching a multiplication lesson (the first in a series of lessons over a six-week period) as they developed their students' understanding of the mathematical concepts associated with the interpretation of the multiplication symbol. An analysis of the findings shows when the teachers used manipulatives, related word problems to the children's lives, and promoted discussion in groups, a greater understanding of multiplication was apparent.

Keywords: Conceptual understanding, numeracy, multiplication, manipulatives

Introduction and literature review

Recent reforms have seen more use of the term *numeracy* in education (Askew, Rhodes, Brown, Wiliam, & Johnson, 1997; Bennison, 2015; Coben 2000; Perso, 2006). Numeracy is described as, "the ability to process, communicate and interpret numerical information in a variety of contexts" (Askew et al., p. 6). Consequently, the current education system supports a mathematics curriculum that emphasises concepts and meanings, rather than rote learning, and promotes integrated, rather than piecemeal usage of mathematical ideas (Howley, Larsen, Solange, Rhodes, & Howley, 2007; Stigler & Hiebert, 2004). The concept of numeracy is closely related to that of functional mathematics, where numeracy is often described as applying mathematics in context (Tout & Motteram, 2006). Therefore, numeracy lessons need to allow students to see the relevance it has to them by making connections between what they are learning inside the classroom and the things they care about in the world around them.

In today's mathematics classroom, concepts are taught first and while procedures are also learnt, it is not without first acquiring a conceptual understanding. *Conceptual understanding* is more than knowing isolated facts and strategies. The student understands the relationship between *mathematical* ideas and has the ability to transfer their knowledge into new situations and apply it to new contexts. The emphasis on conceptual understanding has required a change in teaching style for many teachers, with a shift from the more traditional model that focused on students' proficiencies in reproducing existing solution methods and strategies, to one that encourages students to construct their own meaningful mathematical concepts, through an inquiry-based model (Boaler, 2008).

One of the benefits of emphasising conceptual understanding to students is that they are less likely to forget concepts than procedures, and when conceptual knowledge is gained it can be used to reconstruct a procedure they may have otherwise forgotten. Once conceptual understanding is developed, it becomes conceptual knowledge to sit alongside procedural knowledge (Rittle-Johnson, Siegler, & Alibali, 2001). While conceptual understanding is a prerequisite for students to select appropriate procedures to use when solving mathematical problems, it may later be intertwined with procedural knowledge and the combination is much more powerful than either one alone (Wong & Evans, 2007).

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Developing procedural knowledge at the expense of conceptual understanding has often been cited as part of the reason for poor mathematics proficiency (Davis & Renert, 2014). The procedural approach to teaching was referred to by Skemp (1976) as *instrumental understanding* or rules without reason, while conceptual understanding was known as *relational understanding*. When students are drilled in methods and rules that do not make sense to them, it is not only a barrier for their mathematics understanding, but it also leaves the students frustrated, and with a negative disposition towards mathematics in the long term (Boaler, 2008; Davis & Renert, 2014; Whitenack & Yackel, 2002). However, as Schwartz (2008) has asserted, for teachers to focus on the teaching of mathematics conceptually, they must first have conceptual understanding themselves.

Associated with the importance of teaching conceptual understanding in mathematics, is the use of tools and manipulatives. A tool refers to any object, drawing, or picture, which represents that concept (Suh, 2007; Swan & Marshall, 2010). For example, drawings may be used as a tool for emerging ideas, as sometimes it is difficult for students to think about and understand abstract relationships if relying only on words and symbols. A mathematics manipulative is defined as, "any object that can be handled by an individual in a sensory manner during which conscious and unconscious mathematical thinking will be fostered" (Swan & Marshall, 2010, p. 14). Manipulatives are frequently used in mathematics lessons with the claim that they extend students' learning of mathematical concepts and operations, as they make them more comprehensible (Ma, 2010; Schoenfeld, 2011; Swan & Marshall, 2010; Wright, 2014). Manipulatives can be used to represent the mathematical concepts underlying the procedure, and connections need to be made between the two – the manipulative and the mathematical idea (Carbonneau, Marley, & Selig, 2013; Zevenbergen, Dole, & Wright, 2004). However, simply taking manipulatives, picking them up and using them, will not magically impart mathematical knowledge and understanding (Swan & Marshall, 2010). Appropriate discussion is required alongside the use of manipulatives to make the links to the mathematics explicit or the students may end up with misconceptions. Teachers often require professional development on the incorporation of manipulatives into their teaching, to give insights into how they can assist with children's learning (Stein & Bovalino, 2001).

It is central to the learning that teachers have a discussion with their students following the use of manipulatives so that students can explain their solutions to problems (Gould, 2005a; Ma, 2010; Miheso-O'Connor, 2011). The intention for using the manipulative must be clear and the teacher needs to be aware of what interpretation the students are making of them. If the students do not explain their use of the tools and/or manipulatives, then teachers are in jeopardy of replacing verbal rules and procedures, with rules and procedures for using them. Discussion means that understanding the link between the manipulation of the objects and the related symbolic representation (the mathematical equation), can be established (Ma, 2010; Yackel, 2001). The relationship between the manipulative and mathematical understanding and insights is developed when students use the equipment to construct a model and interpret its meaning. Recent research of Flores (2010) indicated that when using the *Concrete to Representational to Abstract (CRA) model* (manipulatives, to pictures or drawings, to numbers only), students seldom made errors in basic mathematics computation, which resulted in improved confidence and assessment scores.

In developing conceptual understanding, teachers provide working environments and practices that encourage students to work in groups (Vosniadou, 2001). The teacher acts as a co-ordinator providing guidance and support in mathematics content learning, alongside the development of skills that allow the students to work together. Critics of this approach to teaching mathematics, maintain that mathematical rigour is being threatened because students are no longer taught standard methods and they are often wasting time chatting to friends in groups (Boaler, 2008). This has meant that some teachers are afraid to try new ideas and methods in their teaching and have returned to more traditional methods (Boaler, 2008). However, the ability to work together in the mathematics classroom is a skill that needs to be taught (Hunter, 2010). Once achieved, it allows students to help each other and utilise mathematical reasoning when explaining their ideas to others.

This article presents an observation of three teachers as they developed their students' understanding of the mathematical concepts associated with the interpretation of the multiplication symbol. It is appropriate to note here that in English-medium classes in New Zealand schools, as with most other

English-speaking systems, the first number in a multiplication expression represents the multiplier and the second number the multiplicand. Hence, in the expression 4 × 5, the number 4 shows the number of sets (multiplier), while the number 5 is the size of each set (multiplicand). This is the everyday interpretation of the multiplication symbol as 'times', thus 4 × 5 could be interpreted as 4 times 5, or 4 groups of 5, or 5 replicated 4 times (5 + 5 + 5 + 5). It is the elementary idea interpreted by many, that multiplication means so many 'sets of', or 'groups of' (Anghileri, 2006; Haylock, 2010) and is the understanding utilised throughout this article.

The teachers and students involved

The three teachers (non-de plumes used) in this article were teaching the senior classes at their respective schools. Mary taught the Year 7 & Year 8 class and Matt taught the Year 6 & 7 class at one school, while Tina taught a Year 5 & Year 6 class at another school. All three teachers had expressed concern (to the researcher) that there were students in their classes struggling to remember their multiplication basic facts (of times tables up to 10 × 10) and this was preventing them solving problems where knowledge of the multiplication tables was required. This article focuses on the interpretation of the multiplication symbol and is based on the first lesson of a six-week numeracy unit focusing on multiplication and division, where the emphasis was on conceptual understanding alongside procedural application.

Observation of lessons

Matt began his lesson by placing animal strip cards [strips of card showing different animals in groups from one animal on a card to 10 animals on a card] in front of the students. Matt admitted that he had not used manipulatives in his mathematics lessons previously and the students were unfamiliar with the concrete materials. Matt began by saying:

> We haven't used these before. I had to go and get them off Miss [teacher's name]. They are called animal strips… So, these have three on them [held up bunny strips] and these have four on them [held up rhinoceros strips]. (Figure 1)

Figure 1: (a) 'Rhinoceros' and (b) 'Bunny' animal strip cards showing 3 × 4 and 4 × 3 respectively

Following a further talk about the different animal strips, Matt allowed the students to discuss the cards in their groups. He then asked them (working in pairs) to use the equipment to show what three times four (3 × 4) looked like. After a quick observation of the different representations, Matt asked one student to show his constructed interpretation of the equation with cards which showed three groups of four (Figure 1a), while another student showed four groups of three (Figure 1b). Matt then said,

> Tell the person next to you what one you think is correct? When I said that I wanted three times four, which of those two options [Matt pointed to the cards] do you think is correct? Not just the one you think is correct, but why you think it is correct.

Matt left the students to discuss the representations in their groups. Initially, some of the students identified the difference between the two representations, while others said they were the same. After some time Matt brought the students back together and the conversation went as follows:

Matt: Okay, so what have we got? Three groups of four, or four groups of three? Three times four, or four times three? First of all, what is the same about them?

Child: They are the same but just the other way around.

Matt: What is the same about them?

Child: The answer.

Matt: The answer is the same. However, finding the solution isn't the same is it? There is exactly the same number of bunnies as there are rhinoceros. There is exactly the same number, but is there a difference in the way they are set out?

Child: Yes.

Matt: Is it an important difference?

Child: Yes.

Matt: (to the whole class): Hands up if you think it is important. [Glanced around the room] About half the class. Hands up if you think it is not important. [Glanced around the room] Four or five of you.

The students discussed among themselves why it might important to recognise the difference in construction and the general consensus was that if they were asked to show their working in a test they needed to understand the difference. Matt reinforced the times symbol as meaning groups of, such as four groups of six, or eight groups of 20. He then changed the manipulatives to Unifix cubes. He asked the students to construct five groups of four. Some students constructed five time four (5×4) and others made four times five (4×5). A similar conversation was held to that which followed the constructions of three times four (3×4) and four times three (4×3) made earlier with the animal strips. Eventually, most of the students identified that there was a difference in their models, but the total number of cubes was the same. Next, the students were asked to show eight times four and finally eight times five (8×5). For the final task, there was one pair of students who made five times eight (5×8).

Mary introduced her lesson by inviting the students to write in the modelling book (a shared group workbook) what they perceived to be the meaning of multiplication. Mary said, "I just want you to write it down somewhere there [pointed to the modelling book]. Just pop down, your thoughts about what you think multiplication is". The children had the opportunity to explain what it was they had written. Responses included: It is a group full of numbers that you double; a group of numbers that can be used in many ways; using numbers that are hard or easy so that you don't have to count in your head; timesing a number by another number to get a number in your head not using algorithms. Most of the students had some idea about what happened when carrying out the multiplication process but had difficulty expressing it mathematically. Eventually, Mary picked up the response of the student who said, "Timesing a number by another number to get a number in your head."

Mary: Let us think about [simple] numbers like six times four, and four times six. What do we think about when we multiply six times four, or four times six?

Child: Either way you get the same answer.

Mary: You are right. Either way, we get the same answer but are six times four, and four times six the same or different in any way?

After a while, one child said:

Um, different because in six times four you are doubling the fours and in four times six you are doubling the sixes.

At this time, Mary placed a box of Unifix cubes on the floor and asked the students to construct two arrays, one showing six groups of four and another showing four groups of six. Mary asked the students why the different models give the same total number of cubes. She then asked them to model five groups of eight and eight groups of five. She asked, "Which model is the easier to use to find the total number?" After some discussion in their groups, the general agreement was eight times five (8 × 5) was easier to image and calculate than five groups of eight (5 × 8). One child mentioned that this was because it was easier to imagine 5,10,15,20 etc. than 8,16,24… The students modelled 5 × 3 with their cubes and then rotated them to show the opposite representation 3 × 5 (Figures 2 and 3). One student suggested that when solving equations such as one hundred times three (100 x 3), it would be much easier to do three times one hundred (3 x 100). This led to a brief discussion about the Commutative Property of Multiplication (where two numbers can be multiplied in either order), which Mary said she would return to another day.

Figure 2: Cubes showing 5 × 3

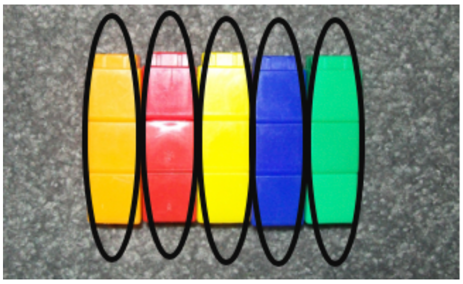

Figure 3: Cubes rotated to show 3 × 5

Tina began her lesson by asking the students what they thought a multiplication equation might look like. Two children had a conversational response:

> Child One: Do you mean something like two plus…
>
> Child Two: [Interrupted Child One]. Plus? That is adding.
>
> Child One: Oh no, it would be (paused)…
>
> Child Two: It is a times table. Two times…
>
> Child One: [interrupted Child Two] So multiplication is a big fancy word for times table?
>
> Child Two: Yep [two times] ... four would equal eight.

Tina reiterated what the children had said, pointed to a large container of Unifix cubes on the floor and asked the children to make (in pairs) what they thought three times five (3 x 5) would look like. Conversations between some of the children included comments:

> So we are going to need eight blocks (children had added the numbers); Three lots of five; That is three fives; Three, three, three, how many threes do we need (groups of three, rather than groups of five)? Five, ten, fifteen; We did it prettily (tried to make the written equation 3 × 5, see Figure 4); Five, five, five.

Tina observed the models and noticed that some had modelled three times five (3 x 5) and some had modelled five times three (5 x 3). She asked each child to record in their books the equation for the representation they had constructed. "Write the number for how many groups you have made (Tina paused while they recorded the number of groups). Now write how many were in each group (paused again)."

Tina then used a context familiar to the students to explain the difference between what the students had modelled and recorded:

> Imagine the blocks (Unifix cubes) are lollies. Over here we have got three bags of lollies with five lollies in each bag (pointed to the recording and modelling of three times five (3 × 5) by one pair of children), and over here we have got five bags of lollies with three lollies in each bag (pointed to the recording and modelling of five time three (5 × 3) by another pair of children).

The students looked at the models and discussed the differences in their pairs. The conversation continued:

Tina: Do we have the same amount on this side [pointed 3 × 5] to as we have on this side [pointed to 5 × 3]?

Child One: Yes, we do.

Child Two: No we don't.

Tina: Do they look the same?

Children: No.

Tina: So, what is the same?

Child One: They both have the same number altogether.

Child Two: They don't look the same, but they equal the same.

Others in the class were still not convinced, so Tina used the names of two children in the class (pseudonyms Alex and Jan) and changed the context to cookies and the problem to division. As Tina explained the problem, the two students were asked to use Unifix cubes to represent the cookies and construct the model.

> Alex and Jan each have 15 cookies in their cupboard. Alex had five people in his family, so how many cookies will each person get? Show me Alex. [Pause while Alex makes five piles with three 'cookies' in each]. Jan has three people in her family. How many cookies will each person get? [Pause for Jan to make three piles with five 'cookies' in each].

The students looked at the Cubes (cookies) and discussed which family they would rather live in. They clearly understood that five people meant three cookies each, while three people meant five cookies each. From here the students recorded the multiplication representation of the two scenarios i.e. five groups of three is 5 × 3, while three groups of five is 3 × 5. The lesson concluded with the students constructing two times six (2 × 6) and six time two (6 × 2). Some students explained the difference in their models (Figure 5). However, there were still a few who appeared unsure of the difference between the two constructs.

Figure 4: One student's representation of 3 x 5

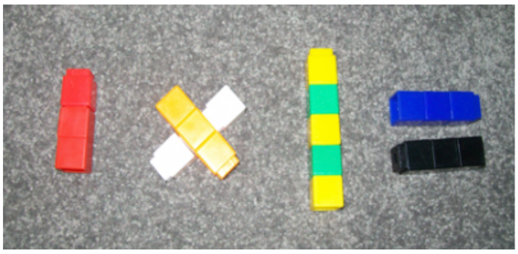

Figure 5: Child explaining (a) 6 × 2 and (b) 2 × 6

Discussion

At the outset of the lesson in each of the three classes, the majority of students had little understanding of what the multiplication symbol represented in an equation and how to accurately model the expression. The teachers wanted to consolidate an understanding of multiplication basic facts up to ten times ten (10 × 10), prior to moving the students on to solving problems involving double-digit multiplication. In order for the understanding to occur, all three teachers utilised a number of similar teaching strategies including the use of concrete manipulatives, written recordings alongside the use of materials, group/pair discussions, and use of real life contexts. One teacher (Tina) also showed a representation of the relationship between multiplication and division to consolidate understanding. Tina used a word problem to dictate the model created (for understanding), by turning the multiplication expressions of three times five (3 × 5) and five times three (5 × 3) into division. It is important that students see the relationship between multiplication and division and the structure of the two problem types (Anghileri, 2006; Clark & Kamii, 1996). The value of simultaneously teaching multiplication and division as 'inverse operations', has been stressed by researchers who have advocated that understanding how these problem structures are connected, can help students generalise when they solve problems later (Ma, 2010; Roche & Clarke, 2009).

The teachers encouraged their students to use manipulatives to consolidate their understanding of multiplication and had sufficient equipment for them all do this simultaneously. Matt's students had not used materials in mathematics before and were introduced to animal cards and Unifix cubes to help explain what the multiplication expressions meant. Manipulatives are frequently used in the numeracy classroom with the claim that they extend students' understanding of mathematical concepts and operations (Ma, 2010; Swan & Marshall, 2010; Wright, 2014) and by the end of Matt's lesson, this was occurring with ease. Mary and Tina's students were more familiar with materials and readily used the Unifix cubes. Some of the students initially struggled connecting explanations between the equations given and the models created. However, as the lesson progressed and further examples were given, the connections between the meaning of the written multiplication symbol and the models became more evident to the students. When manipulatives are utilised to represent the mathematical concepts underlying the procedure, and connections made between the two – the manipulative and the mathematical idea – mathematical understanding becomes greater (Carbonneau et al., 2013; Ma, 2010; Zevenbergen et al., 2004).

The teachers provided many opportunities for discussion among the students, for them to share their thinking, to discuss and justify their ideas. Previous research has shown students need to develop confidence in their ability to think and reason mathematically, and to explain and defend those reasons (Hunter, 2010; Whitenack & Yackel, 2002). It is often in the explanation of a correct answer that a student gains a deeper understanding of a mathematical concept (Kazemi & Stipek, 2001) and when the students shared their thinking with others, it not only allowed them to be sure of their own ideas but allowed their peers to consolidate their thinking also.

When the teachers used stories about lollies and cookies, the students were able to see a connection between their own lives and the mathematics examples. When effective teachers create scenarios and word pictures that appeal to their students, conceptual understanding is acquired by aligning mathematics to their real-life world (Ma, 2010; Miheso-O'Connor, 2011; Mulligan & Mitchelmore, 1997; Schwartz, 2008). Analogies and models are important components of effective explanations, and the ability to transform mathematics ideas through explanations is a necessity for teachers if students are to understand them.

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