Exploring the Mental Strategies of Year 9 Students

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ABSTRACT: This paper reports on a study with eight Year 9 students from a low decile secondary school. Four of the students were high achievers in mathematics (from a mathematics extension class), and four were low achievers in mathematics (from a mathematics applied [MAP) class].

Tasks from the diagnostic interview of the Numeracy Development Project (Numeracy Project Assessment: NumPA) were used to assess the students' mathematics understanding. Additional tasks to explore students' recognition of the connectedness of related mathematics problems were also used.

High achieving students differed from low achieving students in a number of ways, including: the number of questions they could answer, the speed with which they found solutions to the problems, the ease with which they explained their mental strategies, and their overall confidence levels.

Most of the students from both groups expressed a preference for using pen-and-paper, and many of those who used mental strategies simply carried out a formal written algorithm in their heads.

Most students recognised the connection between related mathematics questions and used that information to solve subsequent problems. The only student not to recognise the connectedness of related problems was a low achieving student.

Like many other western countries, New Zealand responded to its poor results on the Third International Mathematics and Science Study (TIMSS) by focusing attention on mathematics learning and teaching in schools, putting a particular emphasis on numeracy (Ministry of Education, 1997, 2001, 2003). The Numeracy Development Project has been a major initiative in mathematics designed to improve the achievement in numeracy of students at every level of the education system. The Literacy and Numeracy Strategy, within which the Numeracy Development Project sits, is guided by several key themes, the first of which is "raising expectations for students' progress and achievement" (Ministry of Education, 2002, p. 1).

The Numeracy Development Project has several key features, including the number framework which consists of a sequence of stages, showing the progression of concepts about numbers and how they can be used (Ministry of Education, 2001, 2003a, 2003b). The framework has two interdependent aspects: Strategy, and Knowledge. Strategies help create new knowledge through use, while Knowledge provides the foundation for strategies. Each strategy stage includes the operational domains of addition/subtraction/place value, multiplication, and fractions/ratios/proportions. Initial stages on the framework are characterised by the use of counting strategies. These become increasingly sophisticated and more efficient until eventually they are replaced by part-whole strategies, which use knowledge of number properties to break numbers apart (partitioning) and put them back together to make calculations easier. Initially, part-whole strategies are acquired within the domain of addition/subtraction, but eventually they are extended to multiplication, and then to problems involving fractions, ratios and proportions. The Numeracy Development Project includes several professional development programmes for teachers: the Early Numeracy Project (ENP) is for teachers at the Year 0-3 level, the Advanced Numeracy Project (ANP) is for teachers at the Year 4-6 level, and the Numeracy Exploratory Study for teachers at the Years 7-10 level. The last consists of an Intermediate Numeracy Project (INP) for teachers at the Year 7-8 level, and a Secondary Numeracy Project (SNP) for teachers at the year 9-10 level.

The Numeracy Development Project puts a strong emphasis on developing students' mental strategies. Mental strategies can be used to simplify calculations by using understanding of number properties and the relationships between numbers and operations. Several mathematics education researchers (e.g., Caney, 2002; Carpenter, Franke, Jacobs, Fennema, & Empson, 1997; Heirdsfield, 2000; Heirdsfield, Cooper, & Irons, 1999; McIntosh & Dole, 2000) have argued that there is a relationship between mental computation and the development of a child's number sense. If children are encouraged to develop their own mental computation strategies then, through this process, it is thought that they:

> learn how numbers work, gain a richer experience in dealing with numbers, develop number sense, and develop confidence in their ability to make sense of number operations. (Heirdsfield, 2000, p. 2)

Mental computation has been defined as:

> computing an exact answer to a computation "in the head." Thus, no external tools, such as a calculator or paper and pencil, are used in doing the computation. The strategy for computing may be invented by the user or "borrowed" from standard paper-and-pencil techniques (McIntosh & Reys, 1997, p. 2),

while number sense is defined as:

> a person's general understanding of numbers and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgments and to develop useful and efficient strategies for dealing with numbers and operations. (McIntosh & Reys, 1997, p. 2)

Although there has recently been a great deal of interest in mental computation as an important computational method, Heirdsfield et al., (1999) point out that this interest is not new, but its significance is now seen in terms of its contribution to the development of number sense as a whole.

A number of writers have argued that the teaching of standard algorithms before a child is ready can be detrimental to their conceptual development (Baroody & Ginsburg, 1990; Carpenter et al., 1997; Gehrke & Biddulph, 2001; Heirdsfield, 2000; Heirdsfield et al., 1999; Kamii & Dominick, 1997; Resnick, 1989). According to Kamii & Dominick (1997), for children who are still unsure of place value, algorithms "unteach" place value, as they focus on the steps of the algorithm rather than try to make sense of the numbers (p. 58). Invented strategies, on the other hand, show more number sense as they are derived directly from the underlying multidigit concepts (Carpenter et al., 1997).

Most research on mental computation strategies has focused on students in the primary years. This is understandable as these are crucial years in terms of children's conceptual development. However, there is a growing emphasis on developing advanced mental strategies in New Zealand. The Numeracy Exploratory Study has put the focus on the learning needs of students in the middle years (Years 7 to 10) of a child's education (see Irwin & Niederer, 2002; Irwin, 2003). However, more research with this age group is needed.

Several writers (e.g., Caney, 2002; Heirdsfield, 2000; McIntosh & Reys, 1997) have noted that students in the middle-grades (Years 7 to 10) prefer to use traditional pen-and-paper algorithms when mentally computing a number problem, and that by the end of primary school, these methods are consistently used by students regardless of their knowledge and ability. However, the flexibility, accuracy and speed with which students employ these standard algorithms, when carrying out mental computations, appear to differ.

Much assessment at the intermediate and secondary school levels uses paper and pencil tests. In order to investigate the mental strategies students use to solve number problems, it is necessary to interview students individually as they work through problems, rather than relying on pen-and-paper test based methods. Task-based clinical interviews provide the opportunity to focus research attention more directly on children's thinking processes rather than on the patterns of correct and incorrect answers (Goldin 2000). Clinical interviews are interactive and allow the researcher to "read the play" as the interview proceeds (Hunting, 1997). The researcher works from an interview protocol, but is able to digress if needed.

The research reported in this paper used task-based clinical interviews to explore the mental strategies used by Year 9 students to solve number problems, and examined the similarities and differences between high achieving and low achieving Year 9 students in mathematics.

METHOD

PARTICIPANTS

Eight Year 9 students were selected from a decile 3 semi-urban high school of approximately 650 students. Four of the students (two girls and two boys) were high achievers in mathematics (H), and four (two boys and two girls) were low achievers (L). The high achieving students were selected from a mathematics extension class. These students had gained scores of at least 80% on the Progressive Achievement Tests (PAT) in mathematics at Year 9, and been identified by their previous school as requiring extension in mathematics. The low achieving students were selected from a "Mathematics Applied" (MAP) class. These students had scored less than 10% on the PAT mathematics test. The eight students are identified in this paper by two letters and a number; for example, HB1 refers to the first high achieving boy, and LG2 to the second low achieving girl.

PROCEDURE

An individual task-based interview was prepared, consisting of 21 questions designed to cover whole number operations involving addition, subtraction, multiplication and division as well as proportions, ratios and percentages (for details, see Appendix). Tasks were taken from the diagnostic interview from the Numeracy Project Assessment (NumPA), developed for the Numeracy Project. Some extra questions involving percentages were added, as well as some questions involving multiplication, and division to explore students' understanding of the connections between questions (for example, 7 x 49, 14 x 49, and 70 x 49; 42 ÷ 7 and 420 ÷ 7).

The students were interviewed individually by the first author in a separate room during their mathematics period. The interview sessions were approximately 30 minutes and were audio-taped for later analysis. The interview was semi-structured to allow the interviewer the opportunity to probe if required. Participation in the interviews was voluntary and every student invited to take part in this research had consented to participate, and given permission for their interview to be taped. Each taped interview was transcribed.

RESULTS

Similarities and differences between the high and low achieving groups of students were examined, and students' responses to questions were organised under several major headings.

PERFORMANCE ON NUMPA TASKS

The performance of the high achieving students on NumPA tasks indicated that they ranged from approximately Stage 6

(Advanced Additive Part-Whole) to Stage 8 (Advanced Proportional Part-Whole), although it was difficult to judge their placement on the Number framework because of their long exposure to standard algorithms. The low achieving students ranged from approximately Stage 4 (Advanced Counting) to Stage 6 (Advanced Additive Part-Whole) on the framework.

Preference for pen-and-paper

Most of the students indicated at some point in their interview that they preferred working out solutions to problems using pen-and-paper, rather than using mental computation strategies. Student HB1's initial reaction was to look for a piece of paper to carry out his calculations. Student HG2 was able to calculate quickly and accurately, but preferred "using paper 'cause it's easier ... you write everything down rather than try to keep everything in your head." Student LG1 used her fingers to trace the calculation process on the desk as she visualised solving the problem on paper. Student LG2 requested paper, stating that "I'm used to doing it on paper and I do it really fast!"

Preference for standard algorithms

Students from both groups who did use mental strategies often just did the pen-and-paper algorithm in their heads.

Student LG1: (394 + 79): after a long pause "473?".
I added it ... 9 onto the 4 is 13, put the 3 below the 9 and the 1 up on the 9. 9 plus 7 plus 1 becomes 7 ... no 17, put 1 up on the 3, add 3 and 1 together and that's 4!

Student HG1: (394 + 79): 473.
Four and 9 makes 13, put the remaining 1 over to next placing, 9 plus 7 plus 1 is 17, put 1 into the 3 placing [or] Round that [79] to the nearest tens and that [39]) to the nearest hundreds and take 7 away.

However, there were noticeable differences between the two groups in terms of speed, flexibility, accuracy and conceptual understanding of the algorithms used.

Student LB1: (394 + 79): after a very long pause, "463".
How I would work it out in my book, 4 plus 9 is 14 ... oh no 13 and carry the 1 ... 9 plus 7 is 16, carry the 1 from there and make it 17 and plus 3 ... 373.

Use of rote learned rules

The "adding zero" rule and the "10%" rule were also consistently applied by most students. Students HB1, HB2 and HG2 were able to link the "adding zero" rule to the multiplication of multiples of tens.

Student HB2: (70 x 49): 3430.
I multiplied the first answer (to 7 x 49) by 10... 7 times 10 is 70.

The low achieving students were only able to connect this rule to the fact than an extra zero was added to the question.

Student LB2: (420 ÷ 7): 70.
I added a zero 'cause they added a zero onto there [pointing to 420]. (Note: his answer to 42 ÷ 7 was 7).

In terms of the 10% rule, the high achieving students linked this rule to the fact that 10% was one-tenth of one hundred and most were able to apply the rule consistently to other situations such as working with 30%.

Student HB1: (10% of $80): $8.
I divided by 10 'cause percent is 100 and 10 is one tenth of it.

Student HG2: (10% of $80): $8.
You divide 80 by 10.. 10% is divide by 10.

Interviewer: What about 20%?
Student HG2: You divide by 5... how many times the number goes into 100.

The low achieving students only knew this as a rule to apply to any 10% type question.

Student LG2: (10% of $80): after a long pause, "$8".
Because 80 divided by 10 is 8.

Recognition of the connection between related questions

All of the students except student LG1 recognised the connection between Q1 (7 x 49), Q2 (14 x 49) and Q3 (70 x 49). The probable reason that student LG1 did not recognise this connection was that she took so long to calculate the answer to the question mentally that she had forgotten the original question. The low achieving students did not consistently recognise the related questions or, if they did, made the wrong link. For example, Student LB1 recognised that there was a connection between Q4 (42 ÷ 7) and Q5 (420 ÷ 7), but applied the wrong rule.

Student LB1: (420 ÷ 7) responded "12".
I doubled the money from that $6.
Interviewer: How did you know to double the money?
Student LB1: It's just 42 but instead of 42 its 420, oh [pause] oh no, that's wrong [long pause] oh, $42 each ... I counted how many 40s in 420 and I just came up with that one.

Prior knowledge of mathematics

There was a difference between the two groups in terms of the number of questions they could answer successfully. The low achieving students were familiar with calculations involving the four basic operations, but had major gaps in their knowledge of ratios, proportions, and percentages. Discussion with these students revealed a lot of emphasis had been placed on the teaching of the four basic operations at their previous Intermediate school. Their use of standard algorithms to solve these types of problems mentally suggests that traditional pen-and-paper algorithms were encouraged in their previous mathematics classes.

Mental strategies

The selection and efficiency of mental computation strategies employed by the two groups of students provided an interesting basis for comparison. The students' mental computation strategies were broken down into three parts: how the two groups of students dealt with mental calculations involving small sums, large sums or products, or other mathematical operations and unfamiliar concepts. Similarities and differences were examined, along with the variety and efficiency of the strategies used by each group.

When solving problems involving small sums, the two groups employed different strategies.

Student LB2: Q21 (196+10) after a long pause responded "206".
I added 10 to 196 ...
Interviewer: Did you count it up, or just add 10?
Student LB2: ... I counted it up ...
Interviewer: What about large numbers?
Student LB2:... I write it down on paper.

Student HG2: responded "206".
19 and [one] 10 is 20 [tens] so it's 206.

In this case, most of the less competent students counted up to find their answer; one student used a part/whole strategy to solve this problem. In comparison, the competent students used either their knowledge of basic facts or number sense to find their solutions.

For larger sums or products and other mathematical operations, differences in the choice of strategies used by each group were noticeable. The low achieving students employed one of three possible strategies to solve problems involving large sums or products and other mathematical operations: counting up or down, using standard algorithm, or a combination of times tables and repeated addition.

a. Counted up or down

Student LB1: (53 – 26) after a very long pause responded "23".
I counted how much more to make it to 53 ...
Interviewer: Did you start with 26 and count up? I noticed you used your fingers.
Student LB1: ... yes.

b. Used standard algorithms

Student LG2: (7 x 49)
Nine times 7 is 63, add over to that side of the 4. Seven times 4 is 28 add 6 and that's 32, oh [pause] 34 oh [pause] ... I'm used to doing it on paper and I do it really fast! ... so 4 nines is 63 ... [long pause] 343!
Interviewer: Do you know of any other way?
Student LG2: Yep ... make that [49] to 50 and times it by 7 to get an estimate and take the 7 off it.

c. Used a combination of times tables and repeated addition

Student LB2: (7 x 49) after a very long pause, "230?".
3 times 49 and then kept adding it on
Interviewer: Did you keep adding on 49?
Student LB2: Yes.

The high achieving students also employed three possible strategies quickly and efficiently to solve a range of number problems: recalling basic facts, using part-whole strategies, or standard algorithms.

a. Recalled basic facts

Students HB1, HB2, HG1 and HG2: (3/4 of 28) responded "21".
I divided the 28 by 4 which is 7, and 7 times 3.

b. Used part/whole strategies (running total)

Student HB1: (403 – 97): 306.
I took 90 off the 403 ... took 3 out of the 7 to get it to 310 ... took 4 off.
Student HB2: (7 x 49): 343.
Seven times 50 and took away 7.

Though both groups made use of the same number of strategies, there was a substantial difference between the groups in the efficiency of the strategies used. This in turn affected the speed and accuracy of the answers given by both groups.

How students dealt with unfamiliar concepts accentuated the differences between the two groups. The low achieving students either gave a random answer without an accompanying reason for their choice, or gave up after a long pause.

Student LG1: (10 balls to make 15 beanies, how many to make 6?): after an extremely long pause, "2?".

The high achieving on the other hand, either searched for patterns and relationships between numbers, or used estimation to find possible solutions.

Student HG1: (21 boys and 14 girls, what percentage are boys?): after a very long pause, "Don't know".
Interviewer: What are you trying to work out in your head?
Student HG1: How do the two numbers fit together ... like multiples of any other numbers ...
Interviewer: Did you find any multiples?
Student HG1: ... 7 ...
Interviewer: Do you always look for multiples
Student HG1:... Yes.
Student HB1: (90% of 40): 30.
I found out where half way was ... 20 questions right is 50% ... 3/4 is 75% which is 30 ... so the answer is between 30 and 40.

The competent students seemed to have enough confidence in their ability to search out possible solutions.

Time

The speed at which the high achieving students processed and answered questions was markedly different from that of low achieving students. The low achieving students were at times very slow and laborious, as the examples given above clearly show. The low achieving students were not able to recall basic facts automatically, and therefore employed inefficient methods to carry our mental calculations. These methods required time and put a strain on short-term memory as the students tried to process the steps and keep track of their calculations at the same time.

Errors

The high achieving students were quick and accurate when calculating number problems involving the four basic operations, and were more inclined to check their answers.

Student HG1: (12 is 2/3 of a number. What is the number?): 18.
I divided 12 by 2 which is 6 ... 6 times 3 is 18 [then] divide 18 by 3 which got 6 ... times by 2 to give 12.

Some errors occurred as a result of undue speed:

Student HB1: (30% of 80): 18.

I went back to the last one ... 3 eights ... [long pause] ...3 times as much so the answer will be 3 times as much.

In a smaller number of cases, errors occurred because of a misconception:

Student HG1: (30% of 80): 26.
[long pause] I divided 80 by 30 and I get 6, and that's 20 left over.

In the above example student HG1 used the logic that if you divide by 10 for 10%, you must therefore divide by 30 if it is 30%.

Inefficient strategies and the time involved in carrying out the calculations involving these methods impacted on the accuracy of the answers given by low achieving students.

There were some systematic errors that appeared to be due to students relying exclusively on rote manipulations of symbols. This was evident in the responses of student LG1:

Student LG1: (70 x 49) [extremely long pause] 363... oh ...um ...3160.

I multiplied it ...put the zero down there [demonstrating with her fingers]. Seven times 9 and 4, added it all together.

Student LG2 also manipulated symbols and consistently used standard algorithms to solve number problems:

Student LG2: (14 x 49) [extremely long pause and no response]

Interviewer: Tell me how you are trying to get the answer?

Student LG2: I'm trying to multiply

Interviewer: What's stopping you from multiplying?

Student LG2: I'm trying to keep the numbers in my head!

Student LB2 used a pen-and-paper algorithm incorrectly:

Student LB2: Q18 (394 +79) [very long pause] 463.

Put that one [394] on top of that one [79]. Four plus 9 is 13, and 9 plus 7 is 16, and one left over, so that's 463.

Student LB1 made conceptual errors:

Student LB1: Q10 (10% of $80): $10 ...

Interviewer: How did you get $10?

Student LB1: From the 10%.

Explanations given by students

There was a marked difference between the two groups in terms of how they expressed their mental strategies verbally. The low achieving group at times had difficulties verbalising their strategies:

Student LB2: Q4 (42 ÷ 7): [very long pause] $7?

I tried to do that dividing thing ...I just guessed.

Interviewer: What's that dividing thing?

Student LB2: ... the one where you put the 42 down the bottom and 7 on top ... how many times 7 goes into 4.

Their explanations were often long winded and they sometimes changed their answer or lost track of their calculations during their explanations.

Student LB1: Q19 (403 – 97): [extremely long pause] 322.

I took 7 away ... 403 takeaway 90 ... put 7 back in ... I took that 3 away from the 7 so it made it 94 ...I took 94 from 400 so 7 takeaway 3 is 3 oh ... so my answer is 323.

The high achieving students generally gave clear and concise explanations, and were able to offer alternative strategies to work out a range of number problems.

Student HG2: Q19 (403 – 97): [long pause] 306.

I made the 97 ... 100 ... took it off the 403 and then added 3.

Confidence level

It was evident that the confidence level of the two groups differed. Low achieving students were often reluctant to attempt questions they perceived were difficult.

Student LB2: Q3 (70 x 49): [no response] Oh! [long pause] I don't know this one.

Interviewer: What's wrong with this question?

Student LB2: It's too big!

Interviewer: Is it going to take too long?

Student LB2: Yeah, and I'll probably make a mistake anyway.

The low achieving students tended to give their answers as a question, and often sought reassurance from the interviewer to check whether their calculation process or answer was correct. Even when the process students went through was good, they still didn't trust their final answers.

Student LB2: (3/4 of 28): [very long pause] 18?.. half of 28 is 14 and I added on 4... oh no it's not 4... oh wait... [pause]... 21? 'cause half of 28 is 14 and I added on 7.

Interviewer: How did you know to add on 7?

Student LB2: 'cause 7 is left over and that will be $28.

On the whole, the high achieving students were quick, accurate and confident in their answers. When given an unfamiliar number problem, they were more inclined to use their prior knowledge to seek patterns and relationships in order to make sense of the question. This in turn provided these students with opportunities to develop their number sense further.

DISCUSSION

On the whole, this study found evidence to support the notion of a relationship between mental computation and the development of a student's number sense (Caney, 2002; Carpenter et al., 1997; Heirdsfield, 2000; Heirdsfield et al., 1999; McIntosh & Dole, 2000). High achieving students seemed to have a wider knowledge base to draw their strategies from, and were able to quickly select the most efficient one to use for a particular problem. Their confidence in their ability to solve number problems mentally, allowed them to take "risks". They tended to be more open to exploring other strategies by looking for patterns and relationships between numbers. It appeared that through this exploration, high achieving students were able to develop good number sense.

Inefficient strategies and the time involved in carrying out the calculations involving these methods impacted on the accuracy of the answers given by low achieving students. Hierdsfield (2000) has explained this problem in terms of the load on short-term memory:

> Mental computation requires concurrent processing and temporary storage of information (holding interim calculations in memory), and retrieval of facts and strategies; that is mental computation is cognitively demanding. Complex calculations can be handled more easily by accessing long-term memory for numerical equivalents and efficient calculative strategies, thus eliminating the need for massive calculations and demands on temporary storage. (p. 6)

Unfortunately, low achieving students appeared to have been mainly exposed to calculations involving the four basic operations and clung to the use of standard algorithms which they had been taught. If they were unable to access this method, they resorted to using counting strategies or gave up. As their strategies tended to be inefficient, it took them a long time to process a number problem, and this delay led to problems holding the previous calculations in their head. Mental computation was cognitively challenging and therefore students needed to develop more efficient mental strategies to lessen the load on their short-term memory storage (Hierdsfield, 2000).

The tendency on the part of low achieving students to make calculation errors inevitably must have affected their confidence levels negatively. It seems reasonable to assume that people with low levels of confidence in their abilities would tend not to explore beyond their comfort zone. It would follow then that low achieving students may have become caught up in a vicious cycle as they continued to cling to standard algorithms and a limited repertoire of strategies. They tended to treat numbers in isolation and rarely sought out number patterns or relationships between numbers (Kamii & Dominick, 1997). According to many writers, the exploration of patterns and relationships in mathematics is crucial to developing number sense and critical thinking (Hiersfeld, 2000; McIntosh & Dole, 2000; McIntosh & Reys, 1997).

A major strength of this project was the method used to gather the data on students' mathematical thinking. The use of the so-called "task-based (clinical) interviews" provided the researcher with the opportunity to delve into students' thinking processes. Although the sample for this study was small, the richness of the data collected far outweighed this disadvantage. Teachers will probably recognise many of the examples of mental strategies given in this paper.

According to Hunting (1997), the three necessary ingredients for successful interviews are significant tasks, a sound pedagogical knowledge base, and good interview techniques. The tasks used in this research were varied. The study was based on a sound pedagogical knowledge base. A wide range of literature was examined and questions from the Numeracy Project (NumPA) were used, as well as some additional tasks designed to explore students' recognition of the connectedness of related mathematical problems. The last ingredient, good interview techniques, was an area of possible concern. The interviewer experienced a problem common to "teachers as researchers" in finding it difficult to resist the urge to ask leading questions in order to "teach" the students. No doubt more experience with this approach to gathering data on students' mathematical thinking would enhance the validity and reliability of the interviewer's technique.

This study focused on Year 9 students, a group in the Year 7 to 10 range for whom there is an urgent need to investigate their use of mental computation strategies. This research indicates that problems with mental computation and conceptual development initially experienced at the primary school level, are still relevant at the secondary level. As teachers, we need to ask ourselves if we really need to teach algorithms to our students. Should we not be encouraging students to develop their own strategies to solve problems? For low achieving students, perhaps the first step to improving their mathematics may be to focus on re-building their confidence in their ability to solve number problems. Somehow we need to "un-teach" the standard algorithms and "re-teach" our students that using their own invented strategies is a valued process. Initially their strategies may be basic, but providing them with opportunities to discuss their strategies with other students should help them to refine and broaden their range of mental strategies. Encouraging a culture within the classroom that values productive discussion should help to nurture our low achieving students. As their confidence in their ability improves, they may be more inclined to take risks, try other strategies, search out patterns and relationships, and in the process develop good number sense.

This study has provided many insights into the mathematical thinking of Year 9 students. However, it has also raised further questions that could be explored in the future. It would be fruitful to interview the same eight students at the end of Year 10 to gauge how well the low achieving students have developed a range of mental strategies. By the end of Year 10 all of the students should have been exposed to problems involving proportions, ratios and percentages. It would be interesting to compare the mental strategies of Year 9 students from a different school (e.g., a middle or high decile school) with those of the students who participated in this study. It would be valuable to investigate how the teaching of standard algorithms affects Year 9 students' conceptual development in mathematics. Comparing the mental computation strategies used by Year 9 students who participated in the Numeracy Project with those of Year 9 students who had not might throw further light on the issue of mental strategies used by students at the secondary level.

Acknowledgements

This paper was based on a research report of a small investigation undertaken by Sandra Umaki as part of a University of Waikato Master of Education paper: *HDC0500: Acquiring Numeracy: How Children's Thinking Develops,* taught by Dr Jenny Young-Loveridge.

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APPENDIX

Tasks used to assess students' mathematical understanding and mental strategies.

Multiplication and Division

Mr Smith owns TJ's appliance store in town.

1. Last month he ordered 7 toasters at $49 each. How much did he spend?
2. This month he decided to order 14 toasters at $49 each. How much will he spend this month?
3. If he bought 70 toasters at $49 dollars each, how much did he spend altogether?

Ms Ford decides to enter the Junior Girls Basket ball team into the Rotorua one-day tournament. She had 7 girls in her team.

1. Ms Ford initially read the entry fee as $42 per team. How much would each player have to pay?
2. Ms Ford had misread her form and found that the entry fee was actually $420. How much would each player have to pay?

Proportions and Ratios

1. What is 3/4 of 28?
2. 12 is 2/3 of a number. What is the number?
3. It takes 10 balls of wool to make 15 school beanies. How many balls of wool does it take to make 6 beanies?
4. There are 21 boys and 14 girls in Mrs Young's class. What percentage of Mrs Young's class are boys?

Decimals and Percentages

Ten years ago a door-to-door salesman was able to earn a commission of 10% for every $80 worth of goods sold. How much would he receive for selling:

1. $80 worth of goods?
2. $160 worth of goods?
3. $200 worth of goods?

Today the salesman would now earn a commission of 30% for every $80 worth of goods sold. How much would he receive for selling $80 worth of goods?

Tracey received a mark of 90% on her last Science test. If there were 40 questions in this test, each worth 1 mark, how many questions did Tracey get right?

1. What is 50% of 24?
2. What is 75% of 200?
3. What is 33 and 1/3% of 600?

Addition and Subtraction

Sandra has 394 stamps. She gets another 79 stamps from her brother. How many stamps does she have?

Her brother has 403 stamps and has given away a total of 97 stamps. How many stamps does he have now?

One of Sandra's stamp albums had 196 stamps in it. If she only needed 10 more stamps to fill her stamp album, how many stamps can this album hold?

Sandra's father bought 53 stamps. He gave 26 to Sandra and the rest to her brother. How many stamps did her brother receive from their father?