The empty number line: A good idea for mathematics?

Ken Carr

"An alternative form of the number line is the empty number line. "

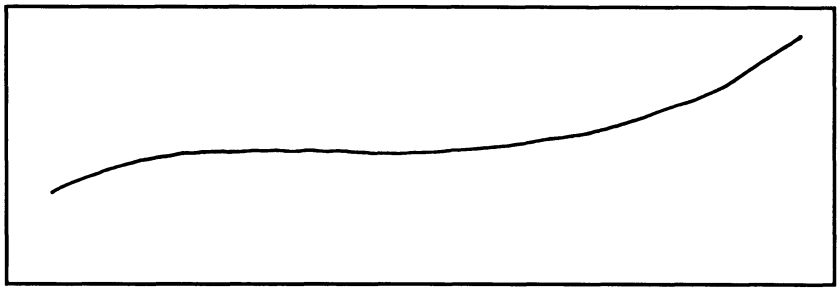
The number line is commonly used by teachers and textbooks in mathematics. These 'conventional' number lines look like a numbered ruler, with straight lines upon which are printed the digits from, say, '1' to '25'. When using this form of number line the students may draw arrows to indicate movement along the line to represent one of the four operations. For example, if the problem were '16 + 4' then the student would start at '16' on the number line and jump four spaces to the right to (hopefully) end up at '20'. Subtraction usually means moving a number of spaces to the left. Sometimes arrows may be put on a number line to represent an operation, and students will be asked to write an equation to match what the arrows show. Nothing could appear more simple.

However, students have not always been able to work successfully with this particular number line model. One of the main problems has been that the students confuse the spaces between the numbers with the numbers themselves. In this case a student starting at '6' and adding on '4' might say "six, seven, eight, nine - six and four is nine". We have observed year 8 students use the 'conventional' number line in this way (Carr and Katterns, 1984).

An alternative form of the number line is the **empty number line**. This idea grew out of disquiet with the model described above, but also from the approach to mathematics education that the Netherlands has taken since the 1970s. The empty number line has become a commonly used tool in mathematics classrooms in Holland.

It works like this. Say students were wanting to work out a problem where they were reading a book having 64 pages. They had read 37. How many more to the end of the book? (In New Zealand classrooms a typical approach might be to make up a subtraction algorithm, and then solve this through using the renaming process by first taking away the ones, then the tens). Many Dutch children do not do this. With the empty number line they first draw a line across the page.

In considering the above problem, the answer can be found by counting on from the smaller number to the larger - this will give you the number of pages left to read. Dutch mathematics educators claim that this counting process is a more natural and realistic way of solving the problem than using an algorithm.



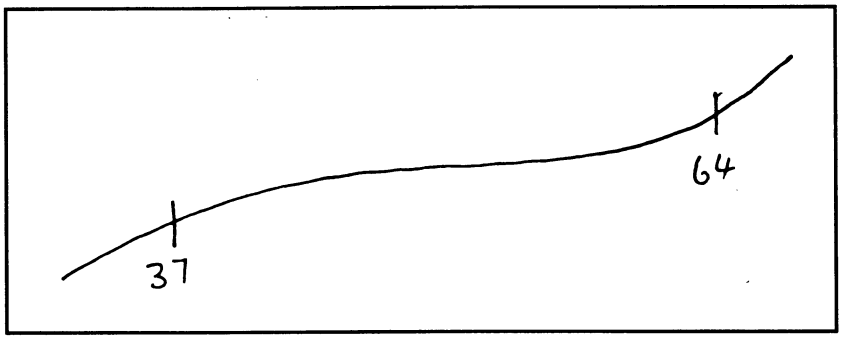
So, lets put the numbers on the number line. We don't have to be exact.

We can estimate where they might go.

The problem is now ready to

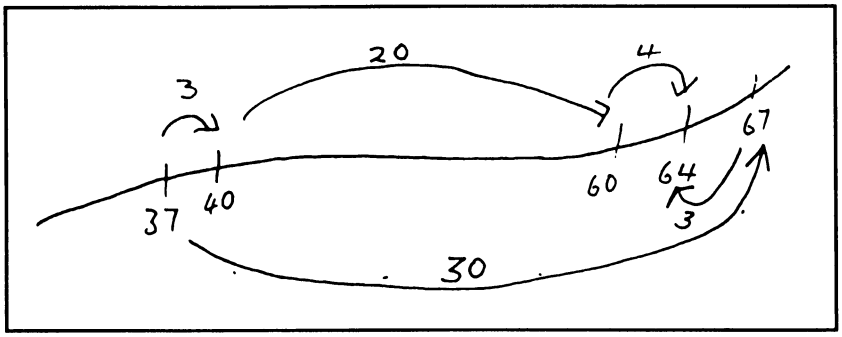
Ken Carr lectures in education at the School of Education, University of Waikato. He has taught mathematics education in the Pacific and in the US. In 1995/6 he studied at the Freudenthal Institute, University of Utrecht, the Netherlands.

solve. This is done by using arrows to show the 'jumps' that are made to get from 37 to 64. On the example below two ways of getting to the answer of 27 are shown.



"The empty number line makes it easier for children to solve maths problems where counting is required. "

What are the advantages of this for students? International surveys (e.g TIMMS) reveal the relatively high mathematical performance of learners from the Netherlands. Their approach to mathematics learning, known as 'realistic mathematics', is used in most schools in that country. Contexts that are familiar to the students (or 'realistic') form the core of what is done. In the primary school operations with numbers are not algorithmic based. In fact the Dutch approach is in line with our curriculum document - students here are encouraged to discover their own ways of solving problems.



"Some claim the empty number line has 'level raising' qualities."

The empty number line makes it easier for children to solve maths problems where counting is required. This is often the case in the primary school. Counting on, or counting back, often does away with the cumbersome, and sometimes unnecessary, formal methods that we as adults impose on learners. For example, is the algorithm using renaming in subtraction and working from right to left a student-generated idea?

Some claim the empty number line has 'level raising' qualities. It has the potential to help students develop more sophisticated strategies as they work out their own creative ways of solving the problem. Contrast this with the formalism and sterile 'rules' for performing subtraction with the place value blocks for instance.

Mathematics educators in the Netherlands are convinced that the empty number line encourages children to try out solutions that make sense to them. As students develop greater skill and understanding, they go on to use processes that are (mathematically) more efficient and powerful.

Reference

Carr, Ken and Katterns, Bob (1984) Does the number line help? *Mathematics in school,* 13, 4, 30-34.